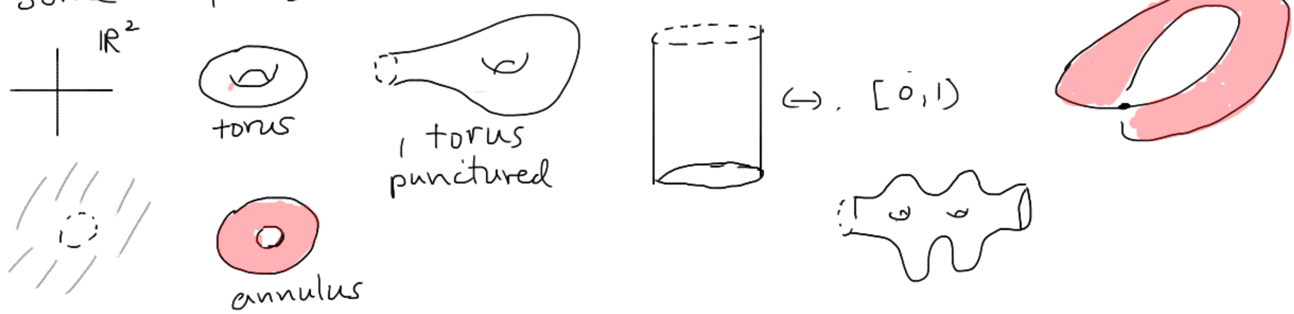
Ch. 2

Some topological surfaces



Interior: the interior of a subset $A \subset X$ is $\mathring{A}$ or $\text{Int}(A)$

$\mathring{A}$ = union of all open sets inside A.

closure: the closure of $A \subset X$ is $\bar{A}$ or $\text{Cl}(A)$

$\bar{A}$ = intersection of all closed sets containing A.

boundary: $\partial A = \bar{A} - \mathring{A}$

Study Lists in Ch. 2!

$$\boxed{\mathring{A} \subset A \subset \bar{A}}$$

Important Properties

① A open $\iff A = \mathring{A}$

② A closed $\iff A = \bar{A}$

Homework

prove ①

$\Rightarrow$ Assume A is open, show $A = \mathring{A}$

$\underbrace{\mathring{A} = \bigcup_{\alpha} U_\alpha \quad U_\alpha \text{ w/ } \underset{\text{open}}{U_\alpha} \subset A}_{\text{def'n}}$. But since A is open $\not\leq$ $A \subset A$

So $A = U_\beta$ for some β so $\mathring{A} = A \cup U_2 \cup U_3 \cup \ldots$
$= A$

$\Leftarrow$ Assume $A = \mathring{A}$.

A is open b/c its the union of open sets by def.

$(\mathbb{R}, F.C.T)$

Let $A = (1, 10)$

$\mathring{A} = \phi$

$\overline{A} = \mathbb{R}$

let $B = \mathbb{R} - \pi$

$\mathring{B} = B$ it's open!

$\overline{B} = \mathbb{R}$