Week 3 – Wed

Problem Set # 2 – Due Monday

Questions ?

1.11

Are these bases?

1. coverage
2. intersections must contain basis elts

(b) $\{[a,b] \mid a<b, a\in\mathbb{R}, b\in\mathbb{R}\}$

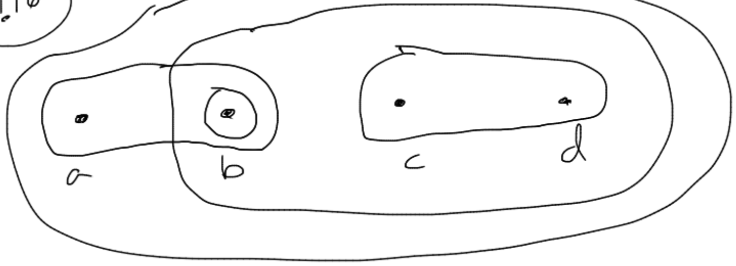
(c) $\{[a,b] \mid a\le b\}$

- $[1,5]\cap[3,7]=[3,5]$
- $[1,5]\cap[5,10]=\{5\}$ ← b/c $\{a\}$ is not of this form.

$\{a\}=[a,a]$

open

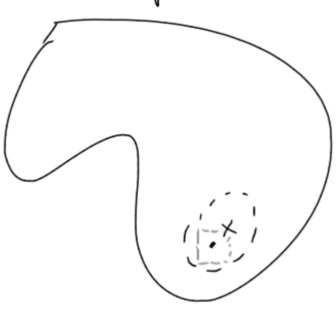
Ex 1.16



Open Sets	Closed Sets (compl. is open)
$\{a,b,c,d\}$	ϕ
$\{\phi\}$	$\{a,b,c,d\}$
$\{b\}$	$\{a,c,d\}$
$\{c,d\}$	$\{a,b\}$
$\{b,c,d\}$	$\{a\}$
$\{a,b\}$	$\{c,d\}$

1.16 HW

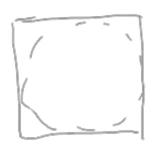
Std. Top



U.

$T_\square \subset T_{std}$

$T_{std} \subset T_\square$



Assume U is open in T_{std}. This means $\forall\, x\in U\ \exists\ B(x,\epsilon)$ for some $\epsilon>0$. To show this is open in $T_\square$, find a open rectangle containing x which is contained in U.

1.31

$\{n\}$ closed $\Leftrightarrow$ $\underline{n \text{ is even}}$

$\Rightarrow$ Assume $\{n\}$ is closed

So $\mathbb{Z} - \{n\}$ is open

case 1 n is odd : Contradict

$\mathbb{Z} - \{n\}$ is open.

$n = 5$.

why is $\mathbb{Z} - \{5\}$ not open?

$\mathbb{Z} - \{5\}$ = union of open sets

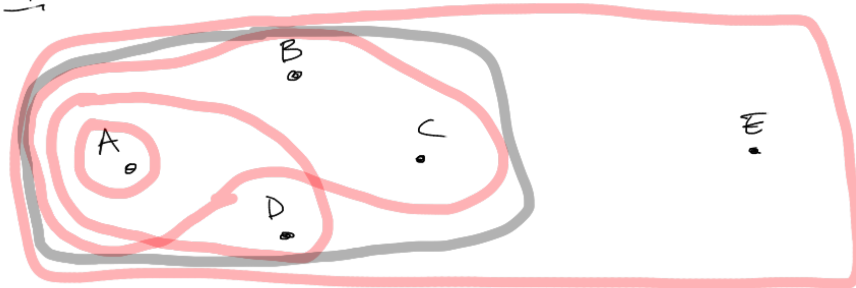
$\Leftarrow$ $\underline{n \text{ is even}}$

odd

$X = \mathbb{Z}$

$T = \left\{ \begin{array}{l} \{\text{odd}\} \\ \{\text{odd even odd}\} \end{array} \right.$

Ex What are the closed sets ?



Is this Hausdorff ?
No
any nbhd of E contains C, (for ex.)

open	closed
$\{A,B,C,D\}$	$\{E\}$
$\{A,D\}$	$\{B,C,E\}$
$\{A\}$	$\{B,C,D,E\}$
$\{A,B,C\}$	$\{D,E\}$
X	ϕ
ϕ	X

not closed
$\{A,D\}$, $\{B,C\}$

$\{B,C,E\}$ not open

not open
$\{B,C\}$

? If $\{x\}$ is closed is it true that every other closed set does contain $\{x\}$. No, $x = \phi$.

Hausdorff Space:

Topological space w/ "Hausdorff property"

given any two points x, y in X

$\exists$ open sets N_x, N_y s.t.

$x \in N_x$, $y \in N_y$ & $N_x \cap N_y = \phi$

$(\mathbb{R}, T_{Std})$ — Hausdorff

given

we can find disjoint nbhds

$(\mathbb{R}, T_{F.C.T.})$ — Not Hausdorff

Let U_1, U_2 be open in $T_{F.C.T}$

$U_1 = \mathbb{R} - \{p_1, \dots, p_n\}$, $U_2 = \mathbb{R} - \{q_1, \dots, q_m\}$

Choose $n+m+1$ points @ least one point lives in both U_1, U_2.