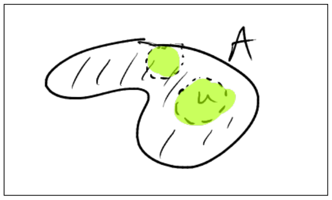
A = any set.

$\mathring{A}$, the interior of A = union of all open sets contained in A

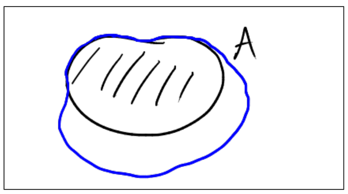
$\bar{A}$, the closure of A = intersection of all closed sets, each containing A.



$\times$ FACT

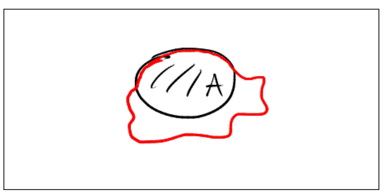
U = open $\Rightarrow$ $U \subset \mathring{A}$

C = closed



$\times$ if $A \subset C$ $\Rightarrow$ $\bar{A} \subset C$

A, B arbitrary sets



if $A \subset B$ $\Rightarrow$ $\mathring{A} \subset \mathring{B}$

$\bar{A} \subset \bar{B}$

thm. A is open $\Longleftrightarrow$ $A = \mathring{A}$

A is closed $\Longleftrightarrow$ $A = \bar{A}$

Proof.

Assume A is open, then $A \subset \mathring{A}$. by def of $\mathring{A}$.

$\mathring{A}$ = union of all open sets inside A, $\therefore$ A contains all unions of all sets inside A,

so $\mathring{A} \subset A$.

$\underbrace{\text{Assume } A = \mathring{A}}$, why must A be open?

b/c it's the union of $^{\text{all}}$ open sets $^{\text{int}}$ b/c that's what $\mathring{A}$ is.

Ex. $A = [2,7)$ in $\mathbb{R}$ with standard topology

$\mathring{A} = (2,7)$ $\quad [3,5] \subset (2,7)$ $\quad (3,5)$ = open, inside A.

$(3,5) \subset \mathring{A}$

$(3,4) \subset \mathring{A}$

$\vdots$

$(2,7) \subset \mathring{A}$

$\bar{A} = [2,7]$

closed, $\Rightarrow$ $\big((-\infty,2) \cup (7,\infty)\big)$ open

contains A.

Intuition = Interior = largest open set contained inside

Closure = smallest closed set containing.

$A = [2,7)$ $\quad (\mathbb{R}$, Discrete Top$)$

open

$\mathring{A} = A = [2,7)$ b/c $\mathring{A} = A \iff A =$ open

$\bar{A} = A = [2,7)$ b/c A = closed b/c $\mathbb{R} - A$ = a set, any set is open in discrete.

$A = [2,7)$ $\quad \mathbb{R}$, finite complement top

$\mathring{A} = \emptyset$ $\quad \mathbb{Q}$

$\bar{A} = \mathbb{R}$. $\quad \mathring{\mathbb{Q}} =$

$\bar{\mathbb{Q}}$