Monday – Week 4

<u>Def'n</u> A subset $A \subset X$ is dense if $\overline{A} = X$

<u>Fact</u>: $\mathbb{Q} = \{ p/q \in \mathbb{R},\ p, q^{\neq 0} \in \mathbb{Z} \}$ is dense in $\mathbb{R}$.

$$\overline{\mathbb{Q}} = \mathbb{R}$$

<u>Theorem</u>: $x \in \overline{A} \iff$ every nbhd of x intersects A.

eg. $\forall\, x \in \mathbb{R} \iff$ every open interval containing it contains a rational.

proceed via the contrapositive:

statement $\longrightarrow$ contrapositive

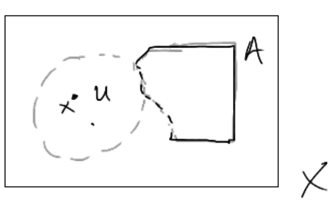
$P \Rightarrow Q \qquad \sim Q \Rightarrow \sim P$

Restating the theorem:

$\exists$ nbhd of x disjoint from $A \iff x \notin \overline{A}$

($\Rightarrow$) Assume $x \in U$, U is open & $U \cap A = \phi$.

Note $U \subset X - A$ (this is equivalent to)



So $A \subset X - U$

1. $x \in U$ so $x \in X - A$
2. $X - U$ is closed. x is <u>not</u> in this set, so x can't be in the closure of A. $\square$

($\Leftarrow$) Assume $x \notin \overline{A}$. There exists some closed set C containing A such that $x \in C$.

$x \notin \underset{\downarrow\ \text{closed}}{\overline{A}} \Rightarrow x \in \underbrace{X - \overline{A}}_{\text{open.}}$ & $\underbrace{X - \overline{A}}$ is disjoint from A is the open nbhd we sought.