Lower Limit Topology

$X = \mathbb{R}$

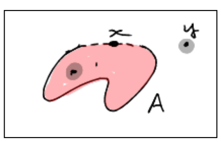
Open Sets $= \xrightarrow{\text{unions}} \{[a,b) \mid a<b\}$

open sets	closed sets
$[a,b)$	$[a,b]$
$(-\infty,b)$	$(-\infty,a]$
$[a,\infty)$	$[a,\infty)$
$(a,b) \leftarrow \bigcup_{i=1}^{\infty} [a+\frac{1}{i}, b)$	$(-\infty,a)$
(a,∞)	$\{a\}$
$\mathbb{R}$	$\mathbb{R}$
ϕ	ϕ
	$[a,b)$

Interior in $\mathbb{R}_{LLT}$	Closure	Boundary: $\bar{A} - \mathring{A}$
$A = \{2\}, \mathring{A} = \phi$	$\bar{A} = A$	$\partial A = \{2\} - \phi = \{2\}$
$B = [1,5], \mathring{B} = [1,5)$	$\bar{B} = [1,5]$	$\partial B = \{5\}$
$C = (1,7], \mathring{C} = (1,7)$,	$\bar{C} = [1,7]$	$\partial C = \{1,7\}$

Limit Points:

For a subset A in X, x is a limit point of A if every nbhd of x intersects A at some point other than x.



Notation: the set of limit points of a set A denoted, A'.

Ex $(\mathbb{R}, \text{std})$

$A = \{5\}$	$A' = \phi$	$\bar{A} = \{5\}$
$B = (1,7)$	$B' = [1,7]$	$\bar{B} = [1,7]$
$C = (2,5]$	$C' = [2,5]$	$\bar{C} = [2,5]$
$D = (-1,1) \cup \{3\}$	$D' = [-1,1]$	$\bar{D} = [-1,1] \cup \{3\}$
$Z = \mathbb{Z}$	$\mathbb{Z}' = \phi \longrightarrow$ (•) • • •	$\bar{\mathbb{Z}} = \mathbb{Z}$
$Q = \mathbb{Q}$	$\mathbb{Q}' = \mathbb{R}$	$\bar{\mathbb{Q}} = \mathbb{R}$

any nbhd of $x = \pi$ intersects $\mathbb{Q}$.

$\frac{1}{2}$ π e

(•) # rationals

Theorem: Let $A \subset X$. $\bar{A} = A \cup A'$.

Ex

$\mathbb{Q} = \{p/q\}$

$\mathbb{Q}' = \mathbb{Q}$

$\bar{\mathbb{Q}} = \mathbb{R}$

$\mathbb{R} = \mathbb{Q} \cup \mathbb{R}$.

since π is a limit pt of $\mathbb{Q}$ you can find a sequence in $\mathbb{Q}$ (seq. of rationals) that get arbitrarily to π