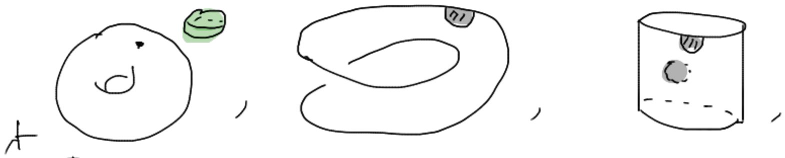
Subspace Topology

Define, precisely, topologies on:



Torus

any subset of a Topological Space.

<u>Idea</u>: How do we think of a subset of (X, T) as a topological space itself. How does it inherit a topology?

Let $A \subset X$, w/ X is a topology.

We define the subspace topology on A by:

The open sets in A $= \{ U \cap A \mid \text{if } U \text{ is open in } X \}$

<u>Ex</u>: $\mathbb{Z}$ = integers is a subset of $\mathbb{R}$. $\mathbb{Z}$ inherits a topology from the std. top on $\mathbb{R}$.

since $\{2\} = \mathbb{Z} \cap (-1, 3)$, $\{2\}$ is open in the subspace topology,

$\{-1, 4\} = \mathbb{Z} \cap \overbrace{(-2, 0) \cup (3, 5)}^{\text{open}}$

The discrete top on $\mathbb{Z}$ is the "standard" top on $\mathbb{Z}$ b/c it is the one inherited from the standard topology on $\mathbb{R}$.

<u>Ex</u> $A = [5, 10)$. $(\mathbb{R}, \text{std})$. What are open sets in the subspace top on A?

$[5, 6) = [5, 10) \cap \overbrace{(4, 6)}^{\text{open in } (\mathbb{R}, \text{std})}$

$[6, 10)$ is not open. No open interval whose intersection with $[5, 10)$ is $[6, 10)$

Ex $Y = [1,4)$. Let Y inherit the subspace topology in $(\mathbb{R}, \text{std})$

open sets in Y

W is <u>open in Y</u> if

$W = Y \cap$ open set in $\mathbb{R}$

$(2,3)$

$[1,2)$

$\phi = Y \cap \phi$

Y open

closed sets in Y.

Def'n: C is <u>closed in Y</u> if

$Y - C$ is open in subspace top
(open in Y)

$[1,3]$ closed b/c its complement in Y is

$(3,4) = Y \cap (3,5)$
open.

ϕ is closed b/c Y is open

Y is closed b/c $Y - Y = \phi$ is open.

$[2,4)$ is closed in Y.

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$[2,4] \cap Y$

Thm: C is closed in $Y \iff C = D \cap Y$ w/ D closed in X.

Let $C, Y \subset X$.