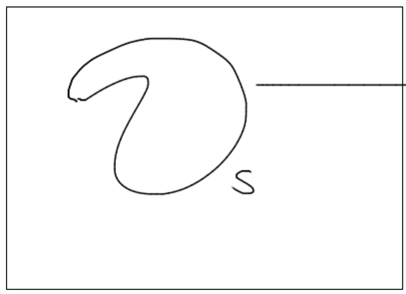
Monday – Week 6

HW #4 – Due Oct. 6 (Wednesday)

Subspace Topology Review:



X

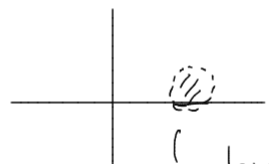
S inherits a topology from X.

Note: If $X = \mathbb{R}^n$ w/ the std. top, then the subspace topology on S <u>is</u> the standard top. on S.

<u>Ex.</u> $X = (\mathbb{R}^2, \text{std})$

$S = \mathbb{R}$

basis elts for X are open ϵ-balls.



intersection is open by def'n of subspace & this coincides w/ std. top on $\mathbb{R}$.

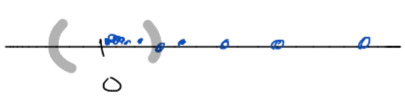
<u>Ex</u> $K = \{\frac{1}{n} \in \mathbb{R} \mid n \in \mathbb{Z}_+\}$. What is the standard topology on K?

$\{1/2\}$ is open, etc. B/c every elt. is open (singletons) this is the discrete

$K^* = K \cup \{0\}$. Show the standard topology here is <u>not</u> discrete.

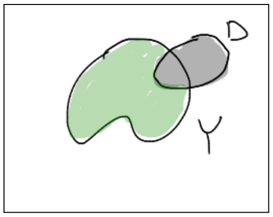
$\{0\}$ is not open (why?) in the std. top on K^* ($\equiv$ subspace top of $K^* \subset \mathbb{R}_{\text{std}}$)

$\{0\} \neq U \cap K^*$ where U is an open interval — any interval containing $\{0\}$ will also contain some $1/m$ for some $m \in \mathbb{Z}_+$.



Thm: Let $Y \subset X$ w/ subspace top.

$Y = [0,1)$ $\mathbb{R}$ std



X

$$C \subset Y \text{ is closed in } Y \iff C = D \cap Y \text{ w/ } D \text{ closed in } X.$$

$C = [\frac{1}{2}, 1)$ is closed in Y

thm $[\frac{1}{2}, 2] \cap Y$ closed

$Y - C = [0, \frac{1}{2}) = (-1, 1/2) \cap Y$ open

proof:

(⟹) Assume $C \subset Y$ is closed in Y.

$Y - C$ is open in Y. So $Y - C = U \cap Y$ w/ U open in X.

$$C = Y - (Y - C) = \underbrace{Y - (U \cap Y)}_{\text{in } Y \text{ not in } U} = (\overset{\text{closed in } X}{\underbrace{X - U}_{\text{not in } U}}) \cap \underbrace{Y}_{\text{in } Y} = D \cap Y$$

w/ $D = X - U$ closed in X.

(⟸) Assume D is closed in X.

$X - D$ is open in X & $Y \cap (X - D)$ is open in Y.

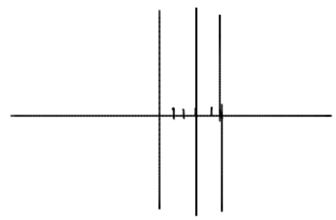
$\underbrace{Y - (Y \cap X - D)}_{= Y \cap D}$ is closed in Y QED

Product Topology

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y) \mid x \in \mathbb{R},\ y \in \mathbb{R}\}$

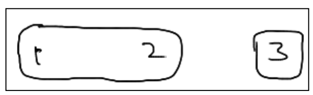
Note: For every x-coord (fixed) you get an entire copy of the 2nd space.

$\mathbb{R}^2$



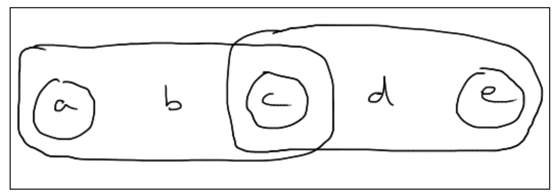
Finite Ex:

$X = \{1, 2, 3\}$,



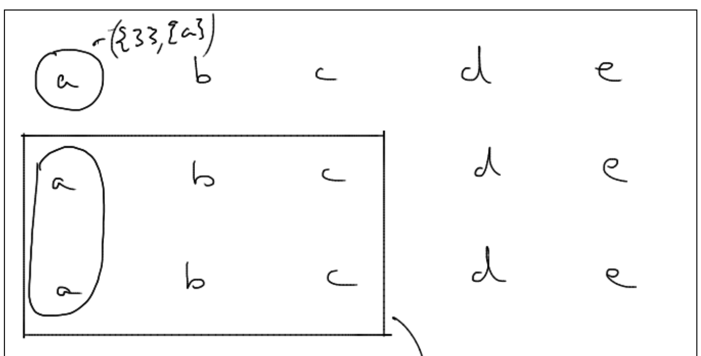
Some Topology on X

$Y =$



some topology on Y

$X \times Y$



open sets in product topology:

any open set in X
$\times$
any open set in Y.

open:

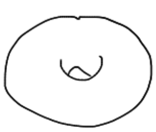
$\{(\{1,2\}, \{a\})$

open in Y

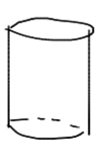
open in X

$(\{1,2\}, \{a,b,c\})$

The following have product topologies



$S^1 \times S^1$



$S^1 \times I$

$[0,1]$

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