Wed – Week 6

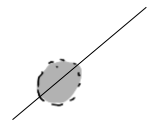
2.26

$\{(x,x) \mid x \in \mathbb{R}\} = D$

$\quad \| $

$\{y = x\}$

$\partial D = \overline{D} - \mathring{D}$.

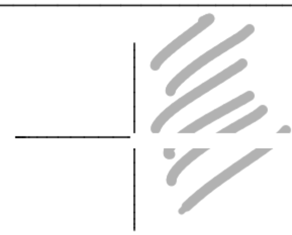


$y = x$

$\mathring{D} = \phi$ b/c $\nexists$ an open set U about any point of D w/ $U \subset D$.

$\overline{D} = D$ ($\Leftrightarrow$ D is closed & this follows b/c $\mathbb{R}^2 - D$ = union of two ½ planes each of which is open.)

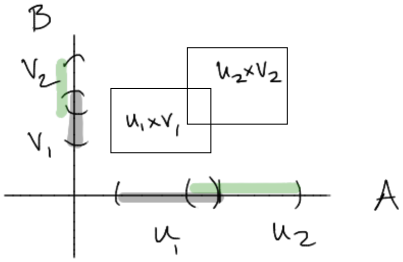
$\{(x,y) \mid x>0, y \neq 0\} = A$



$\mathbb{R}^2$

Product Topology

For topological spaces A, B the product space $A \times B$ has a topology w/ basis: open sets in A $\times$ open sets in B. i.e., a basis for $A \times B$ is $\{U \times V \mid U \text{ is open in } A, V \text{ is open in } B\}$.



Let U_1, U_2 be open in A

V_1, V_2 be open in B

So — a set $C \times D$ is open in $A \times B$ if

$C \times D$ is a union of basis elts (U_α, V_β) w/

U_α is open in A, V_β open in B.

i.e., $C = \bigcup U_\alpha$, $D = \bigcup V_\beta$

each "coordinate-set" is a union of basis elts in its respective topology

Ex. 3.17 If ℓ is a line in the plane, describe the topology ℓ inherits from the product topology

$$\mathbb{R}_\ell \times \mathbb{R}_{std}$$

$\downarrow$ lower limit top $\qquad \downarrow$ std.

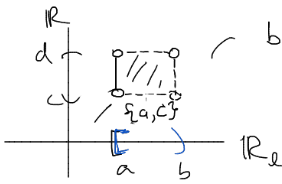
$(a,b)\times(c,d)$

Note: For $\mathbb{R}\times\mathbb{R}$ (std), ℓ inherits the std top

Basis Elts in Std Top on $\mathbb{R}\times\mathbb{R}$

=

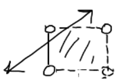
But here $\mathbb{R}_\ell\times\mathbb{R}$



basis elts in $\mathbb{R}_\ell \times \mathbb{R}$

sets of these forms $[a,b)$ & (a,b) will be open

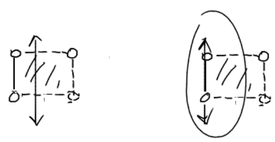
If ℓ has positive slope, both



$\Rightarrow$ lower limit

(c,d)

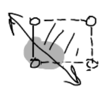
If ℓ is vertical



$\Rightarrow$ Standard top

$\mathbb{R}_{std}$

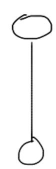
If the slope is negative



$\Rightarrow$ lower limit

$[c+\epsilon, d-\epsilon$

[ ]



$= (c,d)$

$\uparrow$

$\{c+\epsilon, d-\epsilon\}$

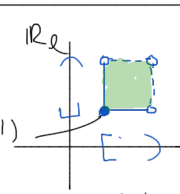
$a, c+\epsilon)$

$(a, d-\epsilon)$

Now repeat for $\mathbb{R}_\ell \times \mathbb{R}_\ell$

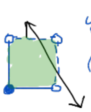
lower limit

$(\pi, -\pi+1)$



Form of basis elts: $[a,b)\times[c,d)$

lower limit



$y=-x+1$ discrete

this point is open b/c

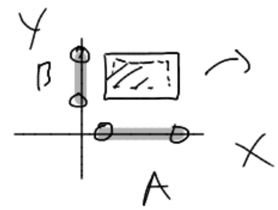
$\{(\pi,-\pi+1)\}$

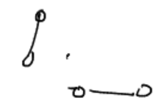
$\{(\pi,-\pi+1)\} = \ell \cap [\pi,4)\times[-\pi+1,17)$

Relationship b/w

(1) interior & product spaces

$$\mathrm{Int}(A \times B) = \mathrm{Int}(A) \times \mathrm{Int}(B)$$





(2) boundary & product spaces