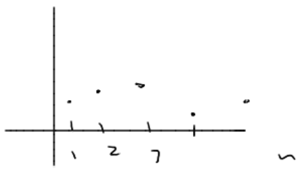
Monday – Week 7

HW 2.19

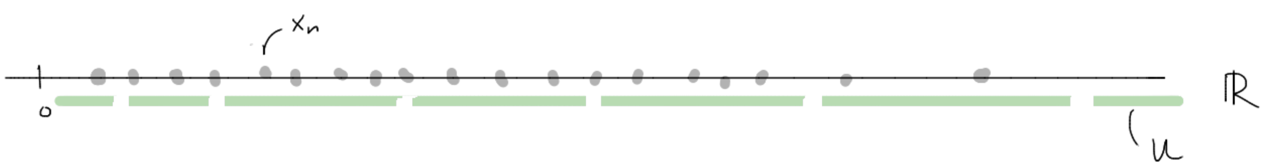
$\{x_n\}$ is an injective sequence in $\mathbb{R}$. this converges to every point in the F.C.T



Let's prove that every point is a limit point of this sequence.

Let $y \in \mathbb{R}$. Let U be an arbitrary open nbhd (in F.C.T.) of y.
(think: $U = \mathbb{R} - \{1\}$ (assuming $1 \neq y$) or $U = (\mathbb{R} - \{1, \dots, 100\} \cup y)$)

B/c the sequence is injective as $n \longrightarrow \infty$ the # of unique numbers in the sequence also goes to ∞. (But U misses only a finite # of them)



28 (a)

∂A is closed, et tal

Hint: Homework problem!

1. $\partial A = \overline{A} - \mathring{A}$ (thm)
2. $\overline{A}$ closed, $\mathring{A}$ open
3.

Tools Needed

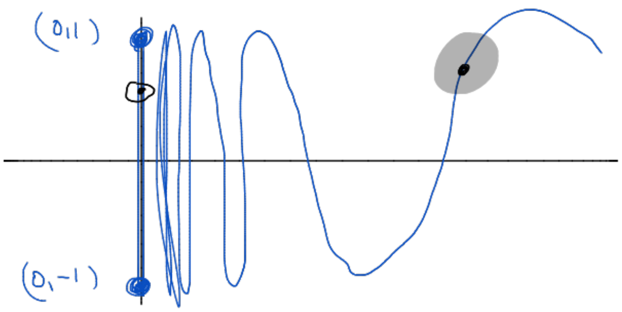
(1) Def'n Boundary:
every open nbhd intersects the set & its complement

(2) Main theorem of boundary.

$$\partial A = \overline{A} - \mathring{A}$$

$\Psi = \{(x, \sin(1/x)) \mid x \in (0,1)\}$

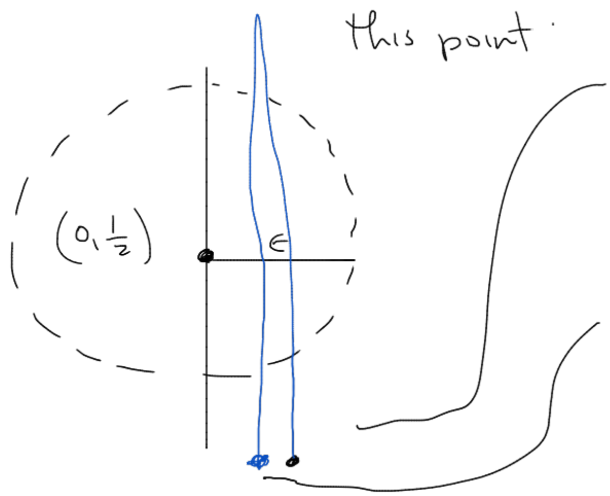
Limit Points:



$\Psi \subset \Psi'$ b/c every $x \in \Psi$ has every nbhd of it intersects Ψ somewhere else.

$L = \{(0, y) \mid 1 \leq y \leq 1\}$

Pick $(0, 1/2) \in L$. We can argue that every nbhd of $(0, 1/2)$ intersects Ψ somewhere.



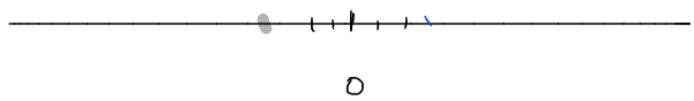
This point: $\left(\frac{\epsilon}{2}, \sin\left(\frac{\epsilon}{2}\right)\right)$

somewhere b/w

$\left(\frac{\epsilon}{2}+\delta, \sin\left(\frac{\epsilon}{2}+\delta\right)\right)$

22 (b)

$x_n = \frac{(-1)^n}{n}$



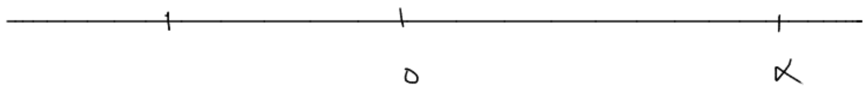
$(-\alpha, \alpha)$

2.22. Consider the sequence defined by $x_n = \frac{(-1)^n}{n}$ in $\mathbb{R}$ with the standard topology.

(a) Prove that every neighborhood of the point 0 contains an open interval $(-\alpha, \alpha)$.

(b) Prove that for each open interval $(-\alpha, \alpha)$, there exists $N \in \mathbb{Z}_+$ such that $x_n \in (-\alpha, \alpha)$ for all $n \geq N$.

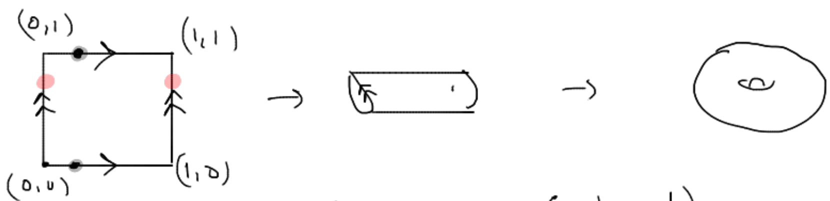
Let $\alpha > 0 \in \mathbb{R}$. (Fix α)



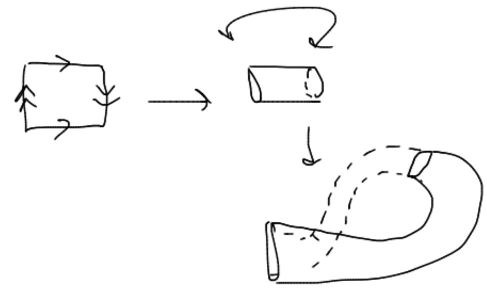
Want $N \in \mathbb{Z}_+$ s.t. $\frac{1}{N} < \alpha \Rightarrow \frac{1}{\alpha} < N$

suppose $\alpha = \frac{1}{1000}$. Your N should be > 1000.

Quotient Topology



begin w/ unit square (closed).



$(0, y) \sim (1, y)$ vs. $(0,y) \sim (0, 1-y)$

$(x,0) \sim (x,1)$

No interior points are glued to any others.