Wed Week 7

ALWAYS

2.24 (g): $\partial A = \bar{A} - \mathring{A}$.

$\bar{A}$ in $(\mathbb{R}^2, \text{std})$, $\bar{A} = A$ sub

$\partial A = A - \mathring{A} = A$

$A = \{(x,0) \mid x \in \mathbb{R}\}$
$= \mathbb{R}$

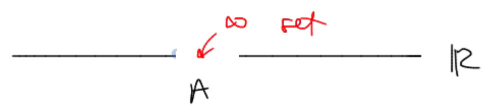
2.15 $A = [0,1]$ in $(\mathbb{R}, \text{F.C.T.})$. Always $\bar{A} = A \cup A'$. What is $\bar{A}$?
Is A closed = Is $\mathbb{R} - A$ open?

NO.

$A \subset \bar{A}$
$\bar{A} = \mathbb{R}$

So $\mathbb{R} = A \cup A'$

$A' = \mathbb{R}$



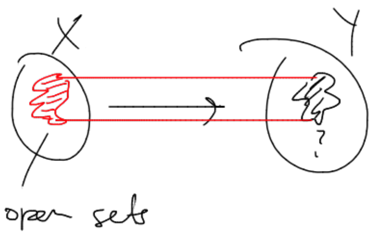
In $\mathbb{R}_\ell$ (a,b) is open

Quotient Topology

<u>Def'n</u>: Given a topological space X, & some set Y, & $f : X \longrightarrow Y$ which is surjective,

The subset $U \subset Y$ is open in the quotient topology if

$$f^{-1}(U) \text{ is open in } X.$$



<u>EX</u>. $X = [0,1]$ $\qquad$ Y = unit circle

$f^{-1}(A) = [0, \pi/3)$

$f(x) = (\cos(2\pi x), \sin(2\pi x))$

$x = 0$, $x = 1$ get glued to $(1,0)$

f maps $[0,1]$ onto ◯

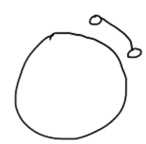
Y has the quotient topology from X

If $A = \{ (\cos(2\pi x, \sin 2\pi x) \mid 0 \le x < \frac{\pi}{3} \}$

$f^{-1}(A) = [0, \frac{\pi}{3})$ is open in std top. on X

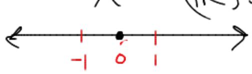
<u>EX</u> $X = \mathbb{R}$ (std top), Y = unit circle, $f(x) = (\cos(2\pi x), \sin(2\pi x))$

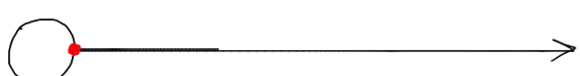
Describe open sets of Y in quotient top.



unit circle

<u>Ex</u> $X = (\mathbb{R}, \text{std})$ $\qquad$ $Y =$

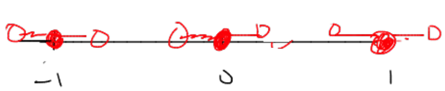


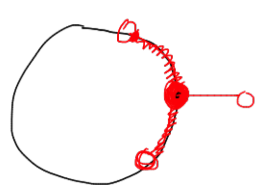


$$h(x) = \begin{cases} f(x) \text{ above} & \text{if } |x| \le 1 \\ (|x|, 0) & \text{if } |x| > 1 \end{cases}$$

Describe open sets in Y

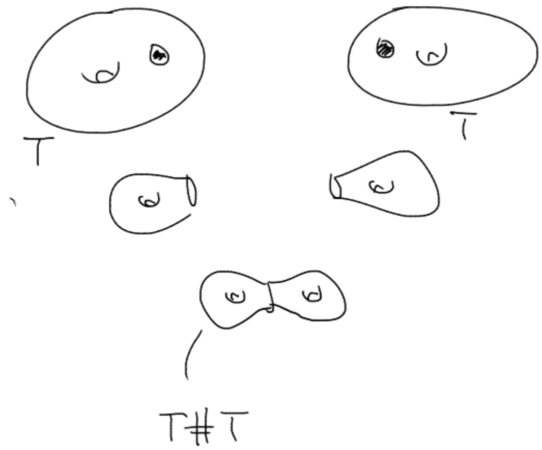
Zoom



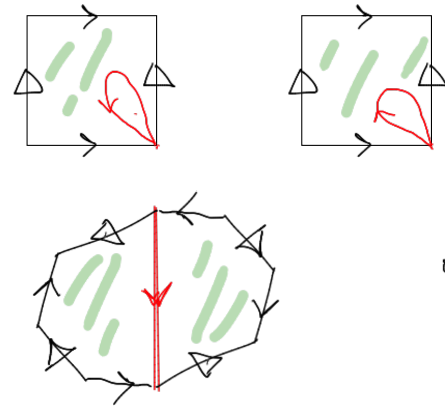


Connected Sum of Surfaces

given two surfaces we create another by
deleting a disk from each & identifying (gluing)
the boundaries



via polygons



octogon