Math 366 - Topology
Midterm
October 12, 2021

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1. Let X be a set. Give the definition of a topology on X.

2. Let X be a set and $\mathcal{T}$ be a topology on X. Give the definition of a basis for $\mathcal{T}$.

3. What are basis elements of the following topologies?
   (a) The finite complement topology on $\mathbb{R}$.
   (b) The lower limit topology on $\mathbb{R}$.
   (c) The standard topology on $\mathbb{R}^2$.
   (d) The discrete topology on $\mathbb{R}$.

4. Determine if the following topologies are Hausdorff. Justify your answer.
   (a) $X = \mathbb{R}$ and $\mathcal{T} = \{\phi, X, (-\infty, p) \mid \forall p \in \mathbb{R}\}$
   (b) The finite complement topology on $\mathbb{R}$.
   (c) The lower limit topology on $\mathbb{R}$.
   (d) The standard topology on $\mathbb{R}^2$.
   (e) The discrete topology on $\mathbb{R}$.

5. Determine whether the following sets are open, closed, both or neither in the given topologies on $\mathbb{R}$.

$$A = (1,2] \ B = [3,4] \ C = \{5\} \ D = (3,5) \ E = [2,5) \ F = \mathbb{R} - \{1,2,3\}$$

   (a) Discrete Topology

   (b) Lower Limit Topology

   (c) Upper Limit Topology

   (d) Standard Topology

   (e) Finite Complement Topology

6. For the sets below, find the interior, closure, limit points and boundary in the given topologies.

$$A = (1,2] \ B = [3,4] \ C = \{5\} \ D = (3,5) \ E = [2,5) \ F = \mathbb{R} - \{1,2,3\}$$

   (a) Discrete Topology

   (b) Lower Limit Topology

   (c) Upper Limit Topology

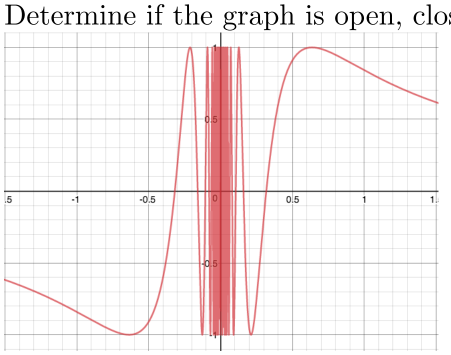
   (d) Standard Topology

   (e) Finite Complement Topology

7. Let X be the graph of $y = \sin\left(\frac{1}{x}\right)$ as shown in the figure below. This means

$$X = \{(x,y) \mid y = \sin\left(\frac{1}{x}\right)\}.$$

   Determine if the graph is open, closed, neither or both in the given topology.



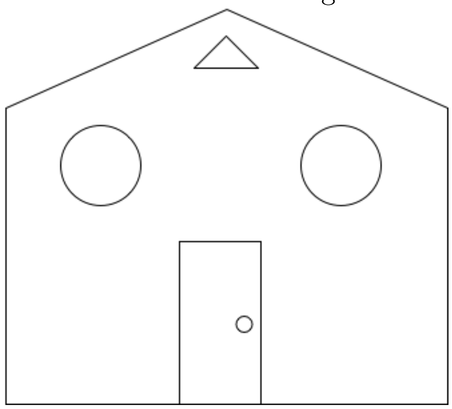
   (a) Discrete Topology on $\mathbb{R}^2$

   (b) Standard Topology on $\mathbb{R}^2$

   (c) Finite Complement Topology on $\mathbb{R}^2$

   (d) Subspace Topology on X

8. Determine if the following set X is open, closed, neither or both in the given topology.



   (a) Discrete Topology on $\mathbb{R}^2$

   (b) Standard Topology on $\mathbb{R}^2$

   (c) Finite Complement Topology on $\mathbb{R}^2$

   (d) Subspace Topology on X

9. This question is all about the set of rational numbers $\mathbb{Q}$.

   (a) What is the interior of $\mathbb{Q}$ in the standard topology on $\mathbb{R}$?

   (b) What is the set of limit points of $\mathbb{Q}$ in the standard topology on $\mathbb{R}$?

   (c) What is the closure of $\mathbb{Q}$ in the standard topology on $\mathbb{R}$?

   (d) What is the boundary of $\mathbb{Q}$ in the standard topology on $\mathbb{R}$?

10. Prove the *Complement-Containment Lemma.*

$$\text{For subsets } A, B \subset X, \text{ if } A \subset B, \text{ then } X - A \supset X - B.$$

11. Show that $\mathcal{T}$ is the discrete topology on X if and only if $\{x\}$ is open for all $x \in X$.

12. Show that for any set $A \subset X$,

$$\overline{X - A} = X - \mathring{A}$$

.

13. Prove that in every Hausdorff space every singleton set is closed.

14. Prove that, in a topological space X, if U is open and C is closed, then $U - C$ is open and $C - U$ is closed.

15. Let $U \subset A \subset X$ and assume U is open. Show

$$\overline{X - int(A)} \subset X - U.$$

Find an example which shows that the two sets are not equal.

16. Show that for a subset $A \subset X$.

$$\partial A \subset A \iff A \text{ is closed.}$$

and

$$\partial A \cap A = \phi \iff A \text{ is open.}$$

and

$$x \in \partial A \iff \text{ if every neighborhood of x intersects A and X-A}$$