Continuity

$f^{-1}(A) = \{x \in \text{domain s.t. } f(x) \in A\}$

$f: X \longrightarrow Y$ is continuous (cts):

if $U \subset Y$ is an open set, then $f^{-1}(U)$ is open in X.

Ex $X = (\mathbb{R}, \text{std})$, $Y = (\mathbb{R}, \text{std})$ $f: X \longrightarrow Y$ by $f(x) = x^2$ is cts.

- $f^{-1}(-1,1) = \{x \in \mathbb{R} \text{ s.t } x^2 \in (-1,1)\}$
  $= (-1,1)$

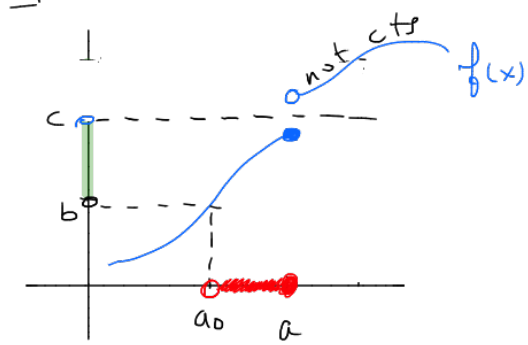
- $\underbrace{f^{-1}(0,4)}_{\text{open in target}} = \underbrace{(-2,0) \cup (0,2)}_{\text{open in domain}}$

- $\underbrace{f(-2,2)}_{\text{open in domain } X} = \overbrace{[0,4)}^{\text{not open in } Y} \implies$ Cts fcns don't always map open sets to open sets.

HW: you'll prove that cts functions:

preimage of closed is closed

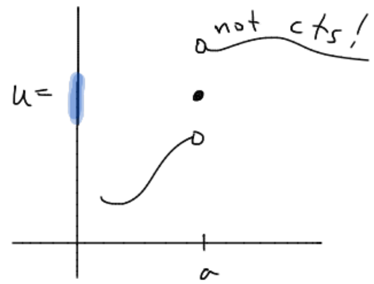
Ex



Let (b, c) be a set in $(\mathbb{R}, \text{std})$ (target)

It's open.

What's the preimage.



$f^{-1}(u) = \{a\}$

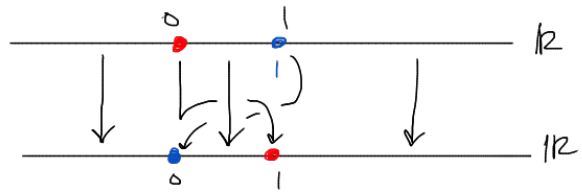
not open

Ex. Every function is cts in discrete top.

Ex $X = (\mathbb{R}, F.C.T.)$

$f : X \longrightarrow X$ by

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \\ x & \text{if } x \neq \{0,1\} \end{cases}$$



f is cts in F.C.T ————

$$f^{-1}(\mathbb{R} - \{2,3,5\}) = \mathbb{R} - \{2,3,5\}$$

$$f^{-1}(\underbrace{\mathbb{R} - \{1,2,3\}}_{\text{open in F.C.T}}) = \underbrace{\mathbb{R} - \{2,3,0\}}_{\text{open in F.C.T}}$$

<u>Note</u>: f is <u>not</u> cts if $X = (\mathbb{R}, \text{std})$

$(-\frac{1}{2}, \frac{1}{2})$ is open in $(\mathbb{R}, \text{std})$

$$f^{-1}(-\tfrac{1}{2}, \tfrac{1}{2}) = \underbrace{(-\tfrac{1}{2}, 0) \cup (0, \tfrac{1}{2}) \cup \{1\}}_{\text{not open}}$$

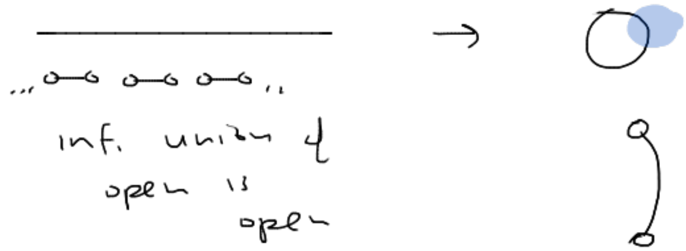
Ex. $f : \mathbb{R}, \text{std} \longrightarrow \mathbb{R}, \text{std}$ Constant Maps are cts

$f(x) = 1$

is cts b/c any nbhd of 1 has pre-image $= \mathbb{R}$ (open)

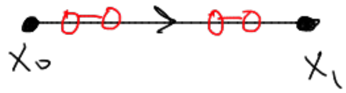
Ex $f: \mathbb{R} \to \mathbb{R}^2$

$f(\theta) = (\cos\theta, \sin\theta)$



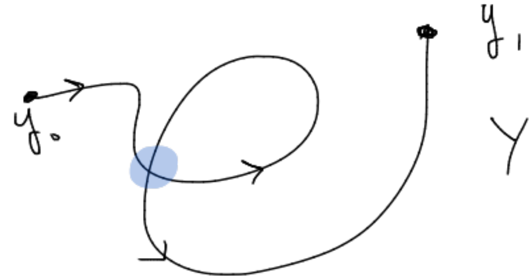
$X \subset \mathbb{R}$

Ex



$f: X \to Y$

cts!



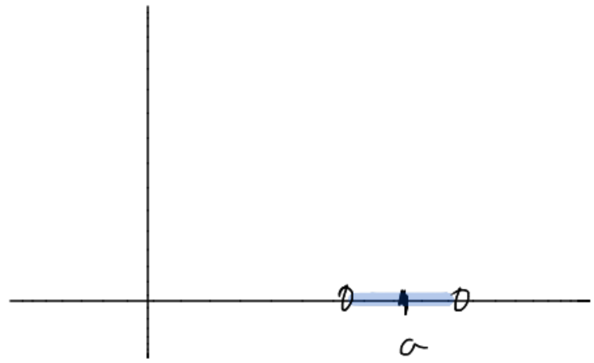
$Y \subset \mathbb{R}^2$

Ex: Multiplication is a continuous fcn.

$\mathbb{R}^2$

$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$

$f(x,y) = xy$



Range

Take open interval about a: (b,c).

What is $f^{-1}(b,c) = \{ (x,y) \in \mathbb{R}^2 \text{ s.t } b < xy < c \}$