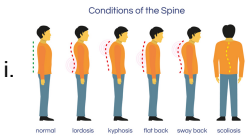
Chapter 7 - Newton

1. A colleague of Newton remarked that, in all his time knowing Newton, he was only seen to laugh once. What made him laugh?
    a. An acquaintance asked, “What good is this old decrepit book of Euclid for anyway?”
2. What did Newton invent that combined his theoretical skill in light and ability to tinker?
    a. reflecting telescope
3. Newton published a precise derivation of the motion of the planets. What is this called, and who persuaded him to publish it?
    a. Principia (natural philosophy principals mathematics) - Edmund Halley
4. Newton first published his theory of fluxions (calculus) where? Hint: in a similar spot where non-Euclidean geometry first appeared.
    a. In an appendix to his treatise on optics.
5. Some people are not appreciated in their time. Newton is not one of these. He became famous at 26 - fellow at Trinity College. He was knighted at age 62. How many more years did he live.
    a. 22. He died at 84 in 1642 + 84 = 1736.
        i. Newton said, “If I have seen farther than most, it is because I was standing on the shoulders of giants.” Discuss this in the following terms:
            1. Interpret this from the point fo view that Newton’s ego was huge and this statement was 400 years old in Newton’s day.
                a. Robert Hooke was diminutive w/ kyphosis
            b.
                i. 

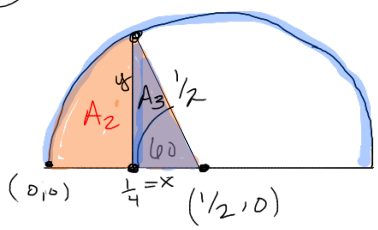
            2. Interpret this sincerely. Give five names for who might these giants be.
                a. Galileo, Euclid, Hooke, Descartes, Archimedes, Pascal, Cardano
6. When was the heroic century of mathematics and who were the heroes?
    a. 17th - 1600s - viete, fermat, pascal, descartes, newton
7. What mathematician did Newton read with great effort? He would read and get stumped. Then he would start over, re-read and get farther, get stumped again. He would go back and read again. This suggests that even the brightest among us have difficulties.
    a. Descartes’ *La Geometrie*
8. True or False. The leading mathematician in Europe largely taught themself (by reading) at Trinity College in Cambridge, England.
9. True or False. Newton frequently gave clear lectures to rooms full of people.

Newton's Approx π

Recall: $(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \dots$

(I)

(II)



Semi-Circle:
center $(1/2, 0)$
radius $1/2$
$(x-\frac{1}{2})^2 + (y-0)^2 = (\frac{1}{2})^2$

$y = \sqrt{\frac{1}{4} - (x-\frac{1}{2})^2}$

$= \sqrt{\frac{1}{4} - x^2 + x - \frac{1}{4}} = \sqrt{x - x^2}$

$= (x-x^2)^{1/2}$

$= \sqrt{x(1-x)}$

$= x^{1/2}(1-x)^{1/2}$

$\cos 60 = \frac{x}{1/2} = 2x$

$\frac{1}{2} \Rightarrow x = \frac{1}{4}$

(III) 3 Regions

Total Shaded Area: $A_1 = \frac{1}{3} \cdot \frac{1}{2} \cdot \pi \left(\frac{1}{2}\right)^2 = \frac{1}{6}\pi \cdot \frac{1}{4} = \frac{\pi}{24}$

(IV) A_3 area: Pyth. thm: $y^2 + (\frac{1}{4})^2 = (\frac{1}{2})^2 \Rightarrow y = \frac{\sqrt{3}}{4} \Rightarrow A_3 = \frac{1}{2}(\frac{1}{4})\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{32}$

(V) A_2 area:

$$\int_0^{1/4} x^{1/2}(1-x)^{1/2}\,dx =$$

(1) use binomial thm to approx this

$$= \int_0^{1/4} x^{1/2}\left[1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots + Cx^8\right]$$

$$= \int_0^{1/4} x^{1/2} - \frac{1}{2}x^{3/2} - \frac{1}{8}x^{3/2} - \dots\,dx = \text{power}$$

w/ 9 terms in expansion he got

__7__ correct decimals of π

How Newton approximated $\sqrt{3}$

$\sqrt{3} = \sqrt{4-1} = \sqrt{4(1-1/4)} = 2\sqrt{1-1/4}$

↑ nearby square (square)

$= 2(1-(1/4))^{1/2}$

$= 2[\star]$

$1 - \frac{1}{2}(\frac{1}{4}) - \frac{1}{8}(\frac{1}{4})^2 - \frac{1}{16}(\frac{1}{4})^3 - \frac{5}{128}(\frac{1}{4})^4$

$= \star$

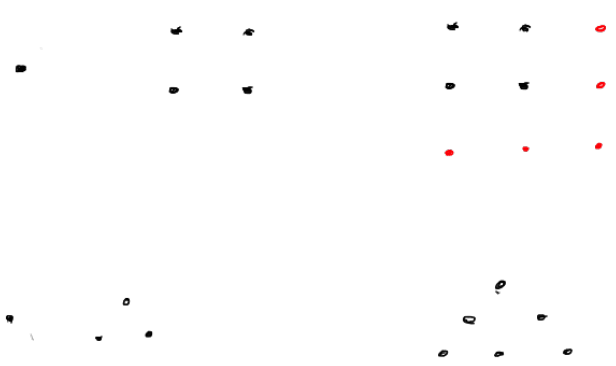
Jacob Bernouilli (1654)

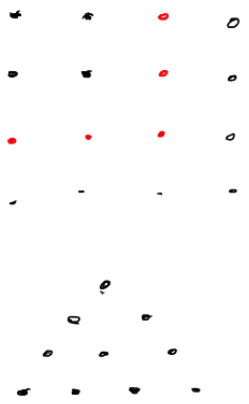
1. The elder by 13 years
2. e = lim (1+1/x)^x
3. Probability
4. Law of large numbers
5. Lemniscate
   a. https://en.wikipedia.org/wiki/Lemniscate_of_Bernoulli

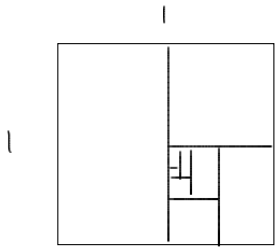
Johann Bernouilli (1667)

1. The younger
2. Calculus / series
3. Competitive / Jealous
   a. After Jacob's death, became jealous of his son Daniel
   b. Falsely dated his work on hydrodynamics to pre-date his son
4. Hired by L'Hospital as tutor
   a. Published 1st book on Calc.
   b. contained L'Hospital's rule
      i. should be Bernouilli's rule

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$







$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \cdots \quad \text{vs} \quad \frac{1}{1} + \left(\frac{1}{2}\right) + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2$$

$= 2$ $\qquad$ $= 1$