Some of Leibniz's Math

1. 1st to explicitly employ the function concept.
   a. tangent, chord, abscissa, ordinate
2. pioneer of actuarial science
   a. he calculated life annuities and liquidation of debt.
3. Gaussian elimination
4. Theory of determinants
5. Cramer's rule: solve systems of equations using determinants
6. $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$
7. Calculus

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

↓ Set $x = 1$

$$\tan^{-1}(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \sum_{k=0}^{\infty} (-1)^k \left(\frac{1}{2k+1}\right)$$

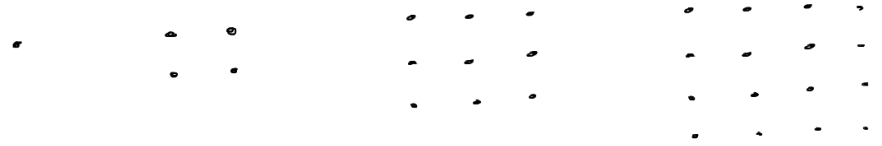
$$\frac{\pi}{4}$$

Leibniz

1. Universal genius
   a. math, philosophy, science, diplomat, theology, ethics, politics, law, history, philology, linguistics, computer sci.
   b. wrote in Latin, French, German + English, Italian, Dutch
2. Rationalist:
   a. 
   b. alphabet of human thought
   c.
   d. 
3. Early life
   a. Father: professor w/ large library
   b. University: 15
   c. PhD: 20
   d. Law degree
   e. Met Christopher Huygens in Paris
      i. 4 years later was mathematical star & left Paris
      ii. Huygens
         1. Dutch Scientist
         2. Huygens studied the cycloid & Saturn's rings
         3. Had pension from Sun King Louis IX
            a. Was King of France from age 4-72
            b. 19+ children with 7 different mothers
            c.
               i. 
         4. Proposed to Leibniz: sum of reciprocals of triangular #'s

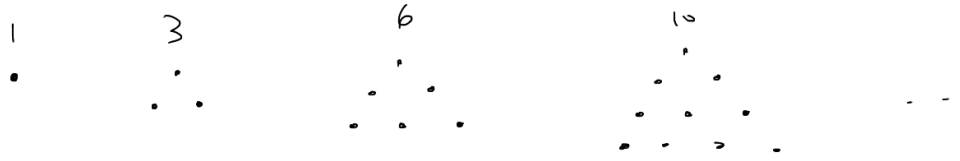
Square Numbers

1 4 9 16



Triangular #'s

1 3 6 10 15 . . .



Sum of Reciprocals of Triangular #'s

$$S = 1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \cdots$$

$$\frac{1}{2}S = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \cdots$$

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right)$$

$$1 + \left(-\frac{1}{2} + \frac{1}{2}\right) + \left(-\frac{1}{3} + \frac{1}{3}\right) + \left(-\frac{1}{4} + \frac{1}{4}\right) + \cdots$$

$$\frac{1}{2}S = 1$$

So, $S = 2$.

$$\boxed{1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \cdots = 2}$$

$\frac{1}{m} - \frac{1}{n} = \frac{n-m}{mn}$

$\frac{1}{6} + \frac{1}{10} - \frac{1}{12}$

$\frac{10-6}{60} = \frac{4}{60} = \frac{1}{15} - \frac{1}{12}$

$\frac{12-15}{}$

$\frac{1}{12} = \frac{4}{12} - \frac{3}{12}$

$= \frac{1}{3} - \frac{1}{4}$

Careful! FAILS. 2 series diverges

$T = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 \cdots$

$= 1 + (-1+1) + (-1+1) + (-1+1) + \cdots$

$= 1$