Viete's Method

$$x^3 + mx = n \qquad \text{sub: } x = y - \frac{m}{3y}$$

$$\left(y - \frac{m}{3y}\right)^3 + m\left(y - \frac{m}{3y}\right) = n$$

$$y^3 - 3y^2\frac{m}{3y} + 3y\frac{m^2}{9y^2} - \frac{m^3}{27y^3} + my - \frac{m^2}{3y} = n$$

$$y^3 - my + \frac{m^2}{3y} - \frac{m^3}{27y^3} + my - \frac{m^2}{3y} = n$$

mult. by y^3

$$y^6 - \frac{m^3}{27} = ny^3 \rightsquigarrow y^6 - ny^3 - \frac{m^3}{27} = 0$$

$$w = y^3 \qquad \underbrace{w^2 - nw - C = 0}_{\text{quadratic}}$$

$$y^6 = w^2 = \frac{n \pm \sqrt{n^2 + \frac{4m^3}{27}}}{2} \qquad y = \sqrt[3]{\frac{n \pm \sqrt{n^2 + \frac{4m^3}{27}}}{2}}$$

Newton

Many of his theorems were great, but his Great Thm was his approx. of π.

Built upon his generalized binomial theorem

Recall: $(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^{\frac{1}{2}} = \sqrt{a+b} = ?$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$\binom{m}{n} = \frac{m!}{n!(m-n)!}$

$P^{\frac{m}{n}}$ $\quad \frac{m}{n}P^{\frac{m}{n}}$ $\quad \frac{m-n}{2n}\frac{m}{n}P^{\frac{m}{n}}$

$$(P+PQ)^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ^2 + \frac{m-2n}{3n}CQ^3 + \frac{m-3n}{4n}DQ^4 + \dots$$

$A, B, C, D, \dots$ are (all of) the immediate preceding terms
coef of Q^i

$$P^{\frac{m}{n}}\underbrace{\left[1 + \frac{m}{n}Q + \frac{\frac{m}{n}\left(\frac{m}{n}-1\right)}{2}Q^2 + \frac{\frac{m}{n}\left(\frac{m}{n}-1\right)\left(\frac{m}{n}-2\right)}{3 \cdot 2}Q^3 + \frac{\frac{m}{n}\left(\frac{m}{n}-1\right)\left(\frac{m}{n}-2\right)\left(\frac{m}{n}-3\right)}{4 \cdot 3 \cdot 2} \cdot Q^4 + \dots\right]}_{(1+Q)^{\frac{m}{n}}}$$

$\frac{\frac{1}{2}-2}{3}$ $\quad \frac{-\frac{3}{2}}{3} = -\frac{1}{2}$

Ex
let $m=1$, $n=2$, $Q=x$

$$(1+x)^{\frac{1}{2}} = \sqrt{1+x} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)x^2}{2} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)x^3}{6} + \dots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$$

One more tool that Newton used to approx π

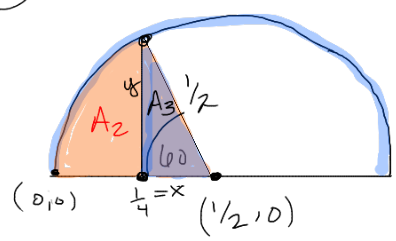
power rule for int $\quad \int 5x^3 + 6x^4\,dx = \frac{5x^4}{4} + \frac{6x^5}{5} + C$

Newton's Approx π

Recall: $(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \ldots$

I

II



Semi-Circle:
center $(1/2, 0)$
radius $1/2$

$(x-\frac{1}{2})^2 + (y-0)^2 = (\frac{1}{2})^2$

$y = \sqrt{\frac{1}{4} - (x-\frac{1}{2})^2}$

$= \sqrt{\frac{1}{4} - x^2 + x - \frac{1}{4}} = \sqrt{x - x^2}$

$= (x - x^2)^{1/2}$

$= \sqrt{x(1-x)}$

$= x^{1/2}(1-x)^{1/2}$

$\cos 60 = \frac{x}{1/2} = 2x$

$\frac{1}{2} \Rightarrow x = \frac{1}{4}$

III 3 Regions

Total Shaded Area: $A_1 = \frac{1}{3} \cdot \frac{1}{2} \cdot \pi \left(\frac{1}{2}\right)^2 = \frac{1}{6}\pi \cdot \frac{1}{4} = \frac{\pi}{24}$

IV A_3 area: Pyth. thm: $y^2 + \left(\frac{1}{4}\right)^2 = \frac{1}{2} \Rightarrow y = \frac{\sqrt{3}}{4} \Rightarrow A_3 = \frac{1}{2}\left(\frac{1}{4}\right)\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{32}$

V A_2 area:

$$\int_0^{1/4} x^{1/2} \underline{(1-x)^{1/2}}\, dx =$$

use binomial thm to approx this