Assume

$$\frac{1}{a}+\frac{1}{a+1}+\dots+\frac{1}{a^2}\ge 1. \quad \text{for any } a \geqslant 1.$$

Starting with 1/a , the sum of the next a^2 - a number
of terms (finite) is greater than 1

$$\frac{1}{1}+\frac{1}{2}+\dots+\underbrace{\frac{1}{a}+\dots+\frac{1}{a^2}}_{\geqslant 1}+\underbrace{\frac{1}{a^2+1}+\dots+\frac{1}{(a^2+1)^2}}_{\geqslant 1}+\underbrace{\frac{1}{(a^2+1)^2+1}+\dots\dots+\frac{1}{((a^2+1)^2+1)^2}}_{\geqslant 1}$$

you can find a endless supply of finite sums exceeding 1.

Induction, limits & partial sums

Leibniz - $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \dots + \frac{1}{\left(\frac{n(n+1)}{2}\right)} = 2$

Here's a more modern proof of this:

$$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{n} + \frac{1}{n+1}$$

$$-S = -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6} - \dots - \frac{1}{n} - \frac{1}{n+1}$$

add

$$0 = -1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \underbrace{\frac{1}{n} - \frac{1}{n+1}}_{\frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}} \qquad + \frac{1}{n+1}$$

$$1 - \frac{1}{n+1} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n+1)}$$

Take Limits $\downarrow$

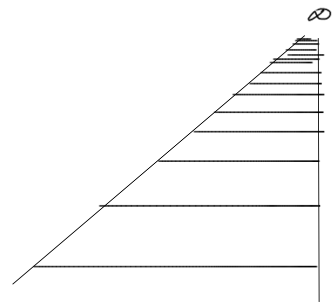
$$1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots$$

So factor $\frac{1}{2}$ out from right side $\quad 1 = \frac{1}{2}\underbrace{\left(1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \dots\right)}$

So this must equal 2.

Induction.

To climb an infinitely tall ladder —



① get on ladder

② Assuming you are standing on the ladder you need to know you climb to the next rung.

That's it.

By the Principle of Math. Induction you can then climb to any rung high up the ladder.

Ex: Show

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \dots = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + = \frac{n}{n+1}$$

① get on ladder, Let $n=1$, $\sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{1\cdot(2)} = \frac{1}{2}$ AND since $n=1$ this equals $\frac{1}{1+1} = \frac{1}{2}$

equal

② $\underbrace{\text{Assume the theorem to be true for } n}$, show it's also true for $n+1$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}, \quad \text{show} \quad \sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n+1}{(n+1)+1} = \frac{n+1}{n+2}$$

③ Expand

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

④ Be creative manipulate equation

$$\underbrace{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)}} = \overbrace{\frac{n}{n+1} + \frac{1}{(n+1)(n+2)}}^{\frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)}}$$

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2}$$

☺