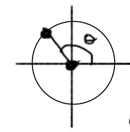
Leonhard Euler: 1707 - 1783

1. Switzerland, Russia, Berlin
2. math / phyics / astronomy / geography / engineer
   a. created graph theory & topology
   b. analytic number theory, complex analysis, calculus
   c. solidified the use of mathematical notation
      i. function notation: f(x)
      ii. greek letter: pi
      iii. imaginary number: i
      iv. summation: Sigma
      v. defined the constant e
      vi. introduced the use of exp function & logs in proofs
      vii. Euler's formula: exp(iz) = cos(z)+isin(z)
      viii. Pioneered analytic methods in number theory
      ix. hyperbolic trig functions
      x. continued fractions
   d. mechanics / fluid dynamics / optics / astronomy / music theory
3. Truly one of the greatest mathematicians in history.

$(4,5)$

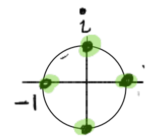
$(\cos\theta, \sin\theta)$ $\mathbb{R}^2$



(a,b)

$e^{i\theta} = \cos\theta + i\sin\theta$

$\mathbb{C}$



①

$a + bi$

$2, 2^2, 2^3, 2^4, \ldots$

i, i^2, i^3, i^4, i^5

$\|$

$(\sqrt{-1})(\sqrt{-1})$

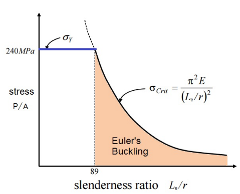
$(-1)^{1/2}(-1)^{1/2} = -1$

$i^3 = i^2 i = -i$

$i^4 = i^3 i = (-i)(i) =$

$= -i^2$

$= -(-1) = 1$



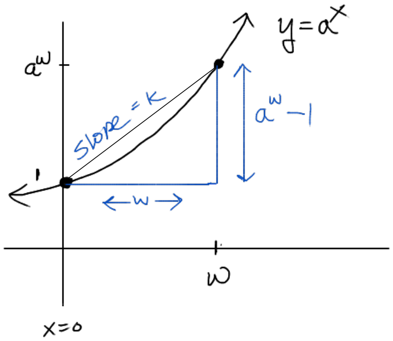
Euler

$$\boxed{e^{i\pi} + 1 = 0}$$

$e \approx 2.1718\ldots$, $i = \sqrt{-1}$

0 , 1 , π

Euler's derivation of the constant e



$k = \frac{\text{rise}}{\text{run}} = \frac{a^w - 1}{w} \xrightarrow[\text{for } a^w]{\text{solve}} \boxed{a^w = 1 + kw}$

Idea of what follows: $18 = \text{Huge \#}_1 \cdot \text{Tiny \#}_2 = 1000 \cdot \frac{18}{1000}$

$\forall x \in \mathbb{R} \quad x = jw \qquad w = \text{wicked small}, \; j = \text{huge} \Rightarrow \boxed{w = \frac{x}{j}}$

Recall: $\lim_{j\to\infty} \frac{j(j-1)}{j^2} = 1$

$a^x = a^{jw} = a^{wj} = (a^w)^j = (1+kw)^j = \left(1 + \frac{kx}{j}\right)^j$

$= 1 + j\left(\frac{kx}{j}\right)^1 + \frac{j(j-1)}{2!}\left(\frac{kx}{j}\right)^2 + \frac{j(j-1)(j-2)}{3!}\left(\frac{kx}{j}\right)^3 + \ldots$ b/c $j =$ huge

$= 1 + kx + \frac{(kx)^2}{2!} + \frac{(kx)^3}{3!} + \frac{(kx)^4}{4!} + \ldots$

(set $k=1 \Rightarrow$ slope $= 1$)
$\Rightarrow$ determines "a"
$a = e$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

sub $x = 1$

$e^1 \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = 2\frac{1}{2} + \frac{1}{6} + \frac{1}{24} = \frac{5}{2} + \frac{1}{6} + \frac{1}{24} = \frac{60 + 4 + 1}{24} = \frac{65}{24}$

$e^1 = e$

$24 \overline{)65} = 2.708\ldots$

In 1730 Euler went out so far to get

$e \approx 2.71828182845904523536028$ — 23 dec. places