MA484 Exam 2 Study Guide - April 13, 2024

1. When & where did each mathematician live?
   (a) Gerolamo Cardano ________________
   (b) Isaac Newton ________________
   (c) Gottfried Leibniz ________________
   (d) Johann & Jakob Bernoulli ________________
   (e) Leonard Euler ________________
   (f) Carl Fredrick Gauss ________________
   (g) Georg Cantor ________________

2. Who Was (Were) the Famous Mathematician(s) That ...
   (a) Was blind? ____________
   (b) Reportedly laughed seldomly? ____________
   (c) Liked to gamble? ____________
   (d) Invented physical devices? ____________
   (e) Is remembered by theorems that are few but ripe? ____________
   (f) Tutored Leonard Euler? ____________
   (g) Had 13 children? ____________
   (h) Befriended Sophie Germain? ____________
   (i) Could compute to 50 decimal places of accuracy in his head? ____________
   (j) Suffered from severe bouts of depression? ____________
   (k) Enjoyed growing vegetables and telling stories their children? ____________
   (l) Had an astounding ability to focus? ____________
   (m) Produced more mathematics than anyone else to this day? ____________
   (n) Never married? ____________

3. Personal Matters

   (a) Describe the Tartaglia / Cardano conflict.

   (b) Describe the calculus controversy in terms of priority, legacy, peronality and nationalism.

   (c) Describe the complicated relationship between the Bernoulli brothers, Leibniz, Newton and L'Hospital.

   (d) Describe any themes or patterns you have noticed concerning *mathematics itself* throughout the history of mathematics.

   (e) Describe any themes or patterns you have noticed concerning the *mathematicians* throughout the history of mathematics.

4. Great Theorems

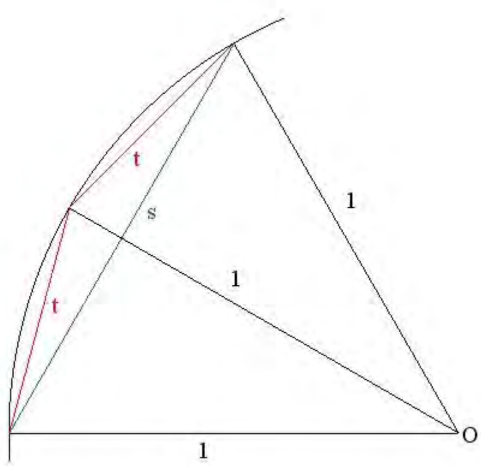
(a) Prove the harmonic series diverges to infinity. Who was the first to prove this?

(b) Prove that, if s is the side of a regular inscribed n-gon and t is the side of a regular inscribed $2n$-gon, then $\sqrt{2-\sqrt{4-s^2}}$. Which of our famous mathematicians would have most likely performed this type of calculation as part of their Great Theorem? (Assume the n-gon and 2n-gon are inscribed in the same unit circle.)

(c) Consider the example l'Hospital gave as the first (!) illustration of his rule in his 1696 *Analyse de Infiniment Petit:*

$$\lim_{x\to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

(a) Verify that if $x = a$, both numerator and denominator are zero. (b) Now use l'Hospital's Rule to determine the limit as x approaches a.



5. Great Problems

(a) Prove by induction that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

and then compute

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

.

(b) Prove that $e^{i\pi} + 1 = 0$.

(c) Suppose that 5 is not a factor of $a, a+1$ or a^2+1. Explain why 5 must be a factor of $a-1$.

(d) Prove that the rational numbers are countably infinite AND prove that the irrational numbers are uncountable infinite.

(e) Begin with the Taylor Series

$$\cos x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

and mimic Euler's work to derive the sum of the reciprocals of squares of odd integers.

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} \cdots = \frac{\pi^2}{8}$$

(f) Write out the first five terms of in the binomial expansion of $\sqrt[5]{1+x}$ and use it to estimate $\sqrt[5]{40}$. (Don't worry you don't have to memorize the generalized binomial formula for the exam).