MA484 Exam 2 Guide - April 16, 2025

1. In the timeframe below list <u>two</u> mathematical **topics** that were discovered/created/studied during this period, **who** was responsible and what **countries** they lived.
   (a) 1500's

   (b) 1600's

   (c) 1700's

   (d) 1800's

2. Personal Matters

(a) The term personal matters includes lots of things, such as interpersonal conflict, ego and fear. Give **3** examples of how these personal matters affected the development of mathematics.

(b) Describe any themes or patterns you have noticed concerning *mathematics itself* throughout the history of mathematics.

(c) Describe any themes or patterns you have noticed concerning the *mathematicians* throughout the history of mathematics.

(d) Describe the Tartaglia / Cardano conflict.

(e) Describe the calculus controversy in terms of priority, legacy, personality and nationalism.

(f) Describe the complicated relationship between the Bernoulli brothers, Leibniz, Newton and L'Hospital.

3. The history of mathematics is full of surprises. Give five mathematical concepts that are surprising or counter-intuitive to you or to those living before you - **and indicate why they are suprising**.

4. (a) State both Fermat's Little Theorem and Fermat's Last Theorem and discuss the place of each within the history of mathematics.

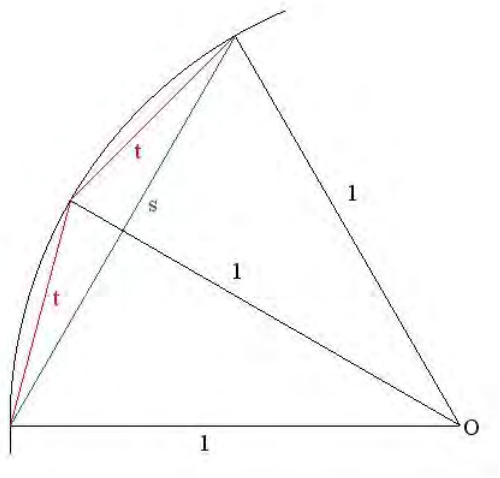
(b) Use induction to prove that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

(c) Use the method Gauss used when he was in **Elementary school** to prove the statement above.

(d) Starting from Newton's definition of the generalized binomial theorem, write out the first five terms of in the binomial expansion of $\sqrt[5]{1+x}$ and use it to estimate $\sqrt[5]{40}$.

(e) Prove that, if s is the side of a regular inscribed n-gon and t is the side of a regular inscribed $2n$-gon, then $\sqrt{2-\sqrt{4-s^2}}$. Which of our famous mathematicians would have most likely performed this type of calculation as part of their Great Theorem?
(Assume the n-gon and 2n-gon are inscribed in the same unit circle.)



(f) Consider the example l'Hospital gave as the first (!) illustration of his rule in his 1696 *Analyse de Infiniment Petit:*

$$\lim_{x \to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$

(a) Verify that if $x = a$, both numerator and denominator are zero.

(b) Now use l'Hospital's Rule to determine the limit as x approaches a.

(g) Begin with the Taylor Series

$$\cos x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

and mimic Euler's work to derive the sum of the reciprocals of squares of odd integers.

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} \cdots = \frac{\pi^2}{8}$$