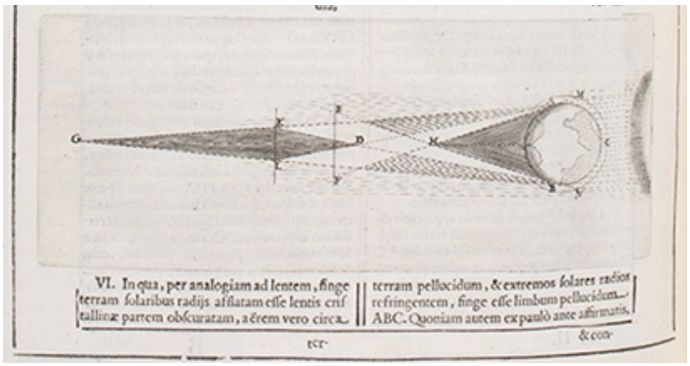
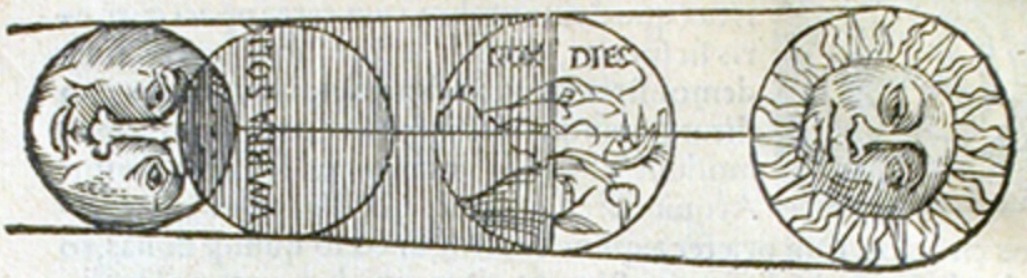
• Johannes de Sacro Bosco's Tractatus de sphaera (1482, 13th-century original)
• Peter Apian's Cosmographia (edited by Gemma Frisius in 1545, 1524 original)
• Mario Bettini's Apiaria universae philosophiae mathematicae (1642)
• Donato Rossetti's Figure de Sistemi del Mondo d'Aristotile Copernico e Ticone (1680)
• John Wallis's Eclipsis solaris Oxonii visae anno aerae Christianae 1654. 2° die mensis augusti (reprinted in Opera mathematica, vol. 2, 1695; 1655 original)

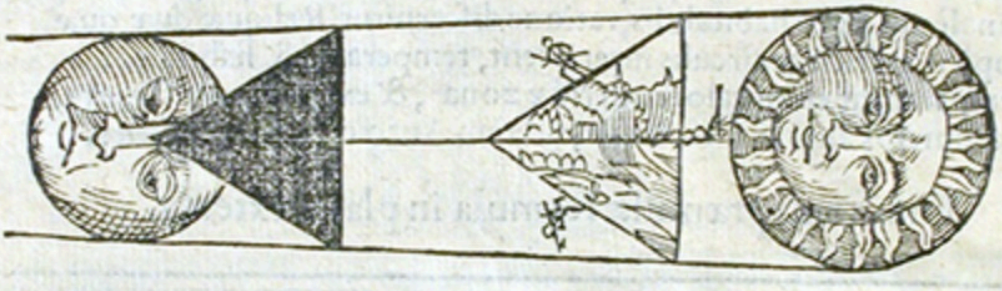
https://maa.org/press/periodicals/convergence/index-to-mathematical-treasures?fbclid=IwAR2hLPYKy2mDBlrTIwHHxFtlSA8UzC_FSKbLyDk8HOh6PMcgRsXeyhOIvkY_aem_AcxkENUK-1YEivu4CMZT-0nXjk63L_hUxV-hH3gjoiDiXCgC_zM90Sa3dQi0l_yz0N1tobCf0h1toNk3ZF74kM5o



Hoc Schema demonſtrat terram eſſe globoſam.



Si terra eſſet tetragona, vmbra quoq; tetragonæ figuræ in eclipſatione lunari appareret.



Si terra eſſet trigona, vmbra quoq; triangularem haberet formulam.

Si terra hexagonæ eſſet figuræ, eius quoq; vmbra in defectu lunari hexagona appareret, quæ tamen rotunda cernitur.

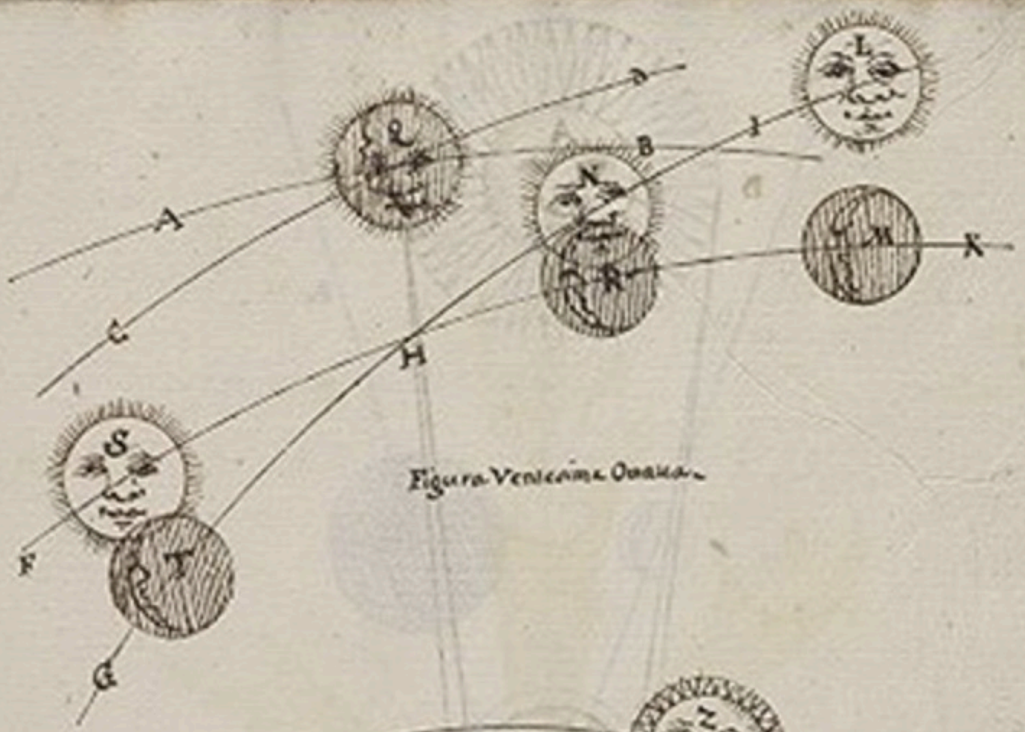


Figura Vencesima Octava.

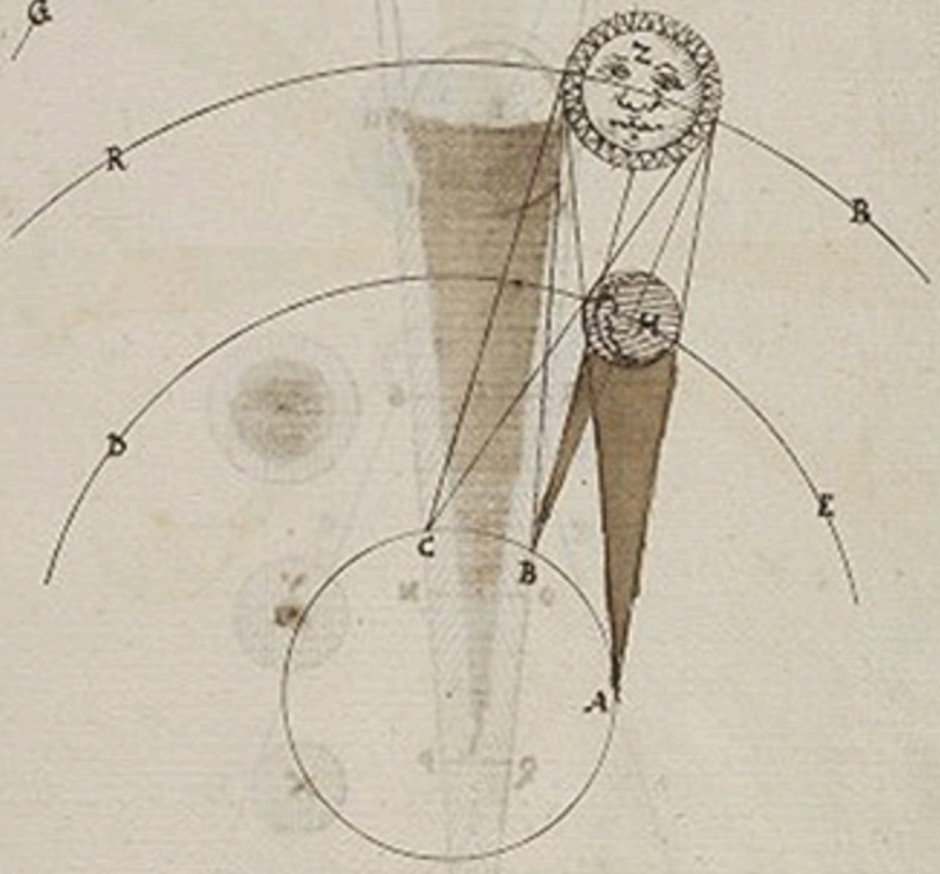
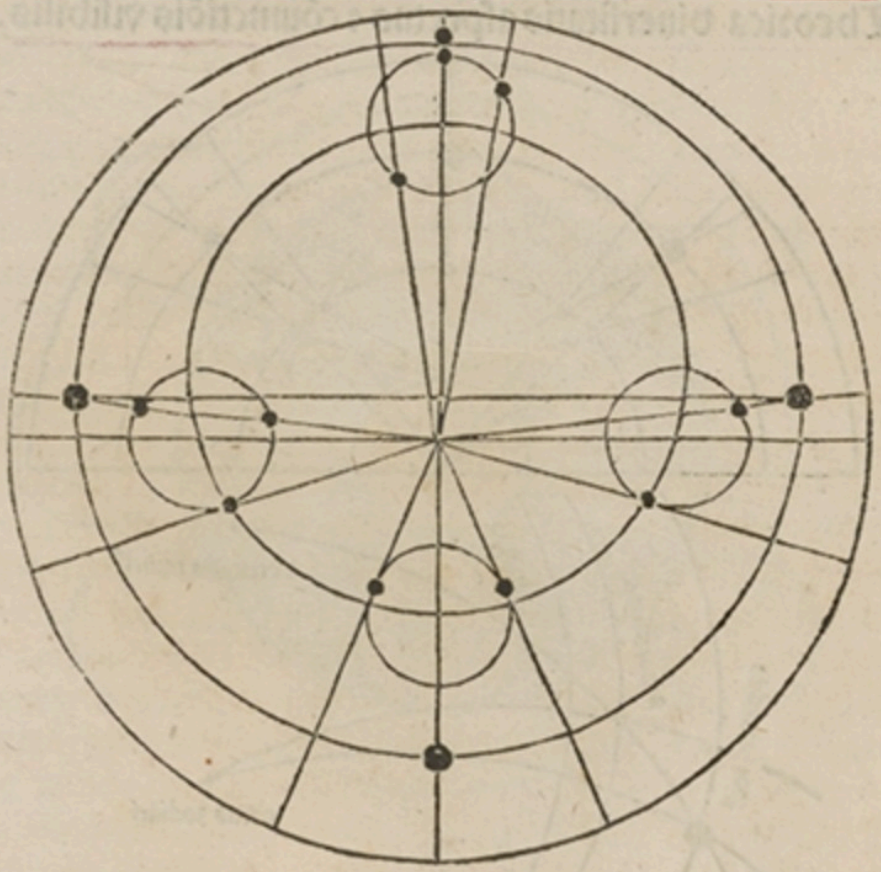


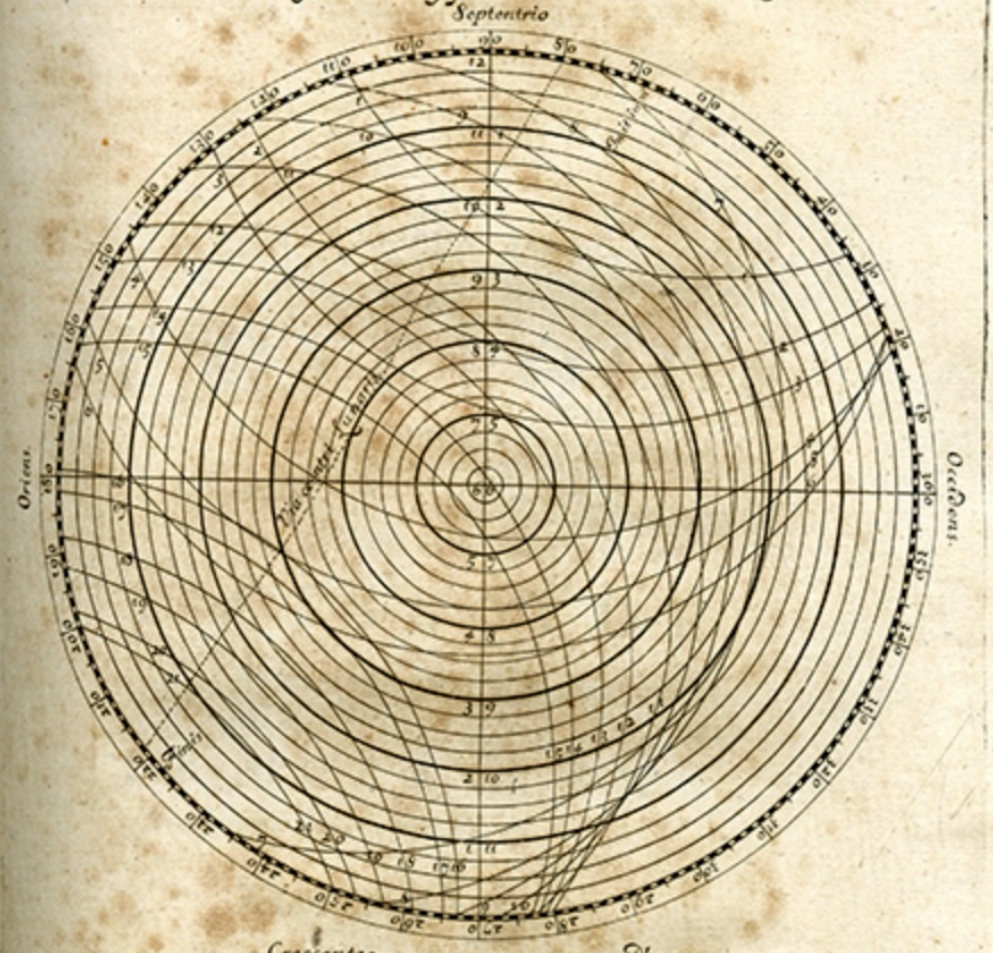
Figura Vigesima Nona.

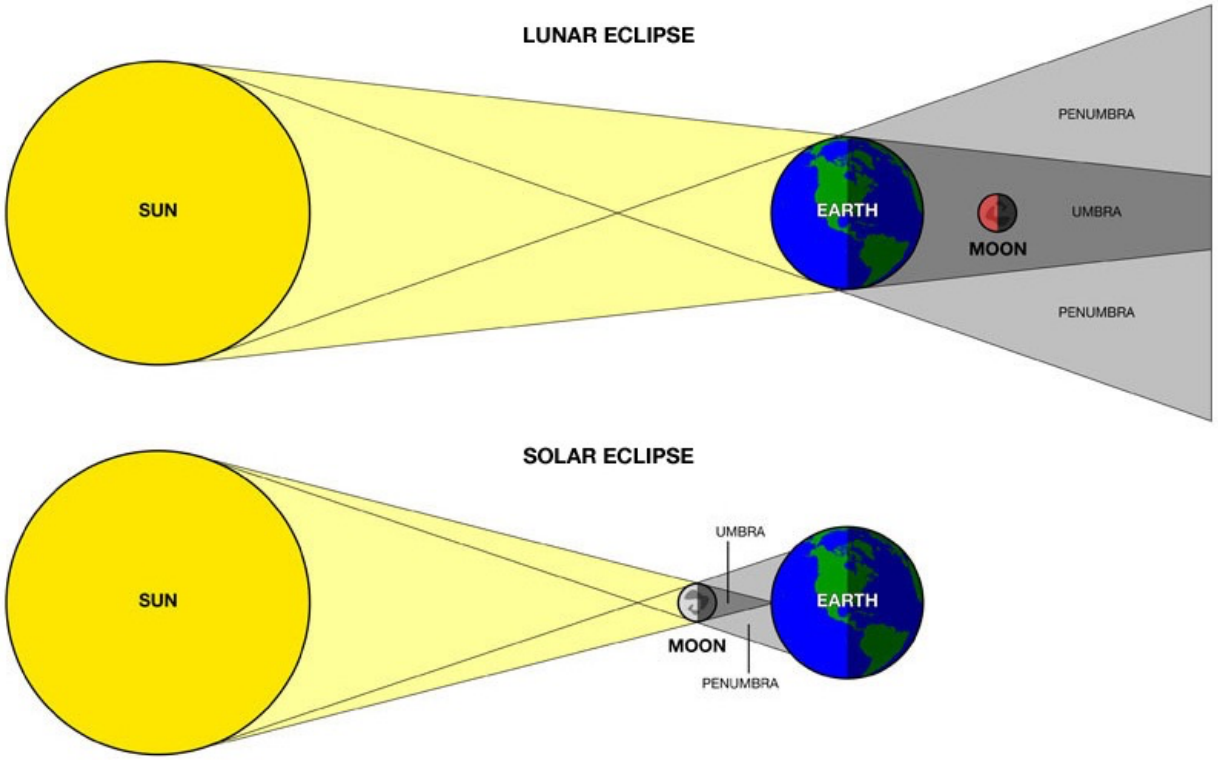
eoꝝ ſcom longitudinē zodiaci cōiūgūt. Vera aūt qñ lineę veroꝝ motuū ſic cōueninnt. Sed viſibilis quādo lineę ab oculo noſtro p centra coꝛpoꝝ ſuoꝝ eductę cōiungunt in vnū. Similit de oppoſitione media ꝛ vera dicendū. Et attendunt hęc in eiſdē ſigno gradu ꝛ minuto. Ex iſto patet ſepe ꝯiūctionē verā cē qñ media pręceſſit aut futura ē. ſępe etiā verā cē qñ tñ viſibilis nō ē. aliqñ etiā viſibilē verā p̄cedere: quādoqz vo ſequi. ¶ Locus verꝰ aſtri ē pūctꝰ ſirmamēti lineā a cētro mūdi p cētꝝ aſtri ꝓtētā terminās. Locus aūt viſus ſiue apparēs p lineā ab oculo p centꝝ aſtri ꝓtractā dcīmiat.

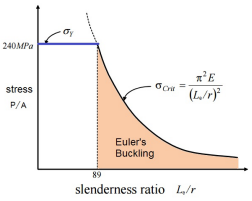
¶ Theorica cōiunctionis ꝛ oppoſitionis luminarium.



Typus Eclipseos Solaris, ut apparuit OXONIJ 2 Augusti, 1654. ante meridiem.




LUNAR ECLIPSE
SUN
EARTH
MOON
PENUMBRA
UMBRA
PENUMBRA
SOLAR ECLIPSE
SUN
UMBRA
EARTH
MOON
PENUMBRA



Leonhard Euler: 1707 - 1783

1. Switzerland, Russia, Berlin
2. math / phyics / astronomy / geography / engineer
   a. created graph theory & topology
   b. analytic number theory, complex analysis, calculus
   c. solidified the use of mathematical notation
      i. function notation: f(x)
      ii. greek letter: pi
      iii. imaginary number: i
      iv. summation: Sigma
      v. defined the constant e
      vi. introduced the use of exp function & logs in proofs
      vii. Euler's formula: exp(iz) = cos(z)+isin(z)
      viii. Pioneered analytic methods in number theory
      ix. hyperbolic trig functions
      x. continued fractions
   d. mechanics / fluid dynamics / optics / astronomy / music theory
      i. Integraded Leibniz's differential calculus with Newton's Fluxions
      ii. nature of & orbits of comets
      iii. foundations of longitude tables
      iv. Optics: foundations wave theory of light, (like Huygens)
      v. Structural engineering: Euler's critical load
      vi. Logic: Euler diagrams (before refinement to Venn diagrams)
3. Truly one of the greatest mathematicians in history.
   a. Laplace: "Read Euler, read Euler he is the master of us all.
   b. Gauss: "The study of Euler's works will remail the best school for the different fields of mathematics, and nothing else can replace it.
   c. Most prolific
      i. 850+ publications
      ii. 92 volumes
   d. Graph Theory / Topology: Seven Bridges of Konigsberg
   e. Basel Problem:
   f. Topology: Euler Characteristic

$(4,5)$

$(\cos\theta, \sin\theta)$ $\mathbb{R}^2$

(a,b)

$e^{i\theta} = \cos\theta + i\sin\theta$

$1+0i$

$a+bi$

$2, 2^2, 2^3, 2^4, \ldots$

i, i^2, i^3, i^4, i^5

$(\sqrt{-1})(\sqrt{-1})$

$(-1)^{1/2}(-1)^{1/2} = -1$

$i^{-3} = i^2 i = -i$

$i^4 = i^3 i = (-i)(i) = -i^2 = -(-1) = 1$

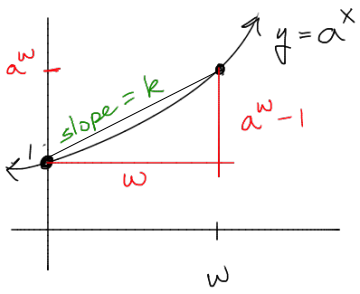
Euler

$e^{i\pi} + 1 = 0$

$e \approx 2.1718\ldots$, $i = \sqrt{-1}$

0, 1, π

Euler's derivation of the constant e



w/ k = slope $= \frac{\text{rise}}{\text{run}} = \frac{a^w - 1}{w}$ so solve for a^w

$kw = a^w - 1 \implies \boxed{a^w = 1 + kw}$

Idea + what follows:

If I choose any #, 12, I can write it as

$12 = 1000 \times \frac{12}{1000}$ = Huge × Small

$\forall x \in \mathbb{R}$ write $x = jw$ where $\frac{x}{j} = w$

$\boxed{w = \text{very small}, \quad j = \text{very large}}$

$$a^x = a^{jw} = a^{wj} = (a^w)^j = (1 + kw)^j = \left(1 + \frac{kx}{j}\right)^j$$

$$= 1 + j\left(\frac{kx}{j}\right) + \frac{j(j-1)}{2!}\left(\frac{kx}{j}\right)^2 + \frac{j(j-1)(j-2)}{3!}\left(\frac{kx}{j}\right)^3 + \cdots$$

Now stand on Newton's shoulders & expand

Recall from Calc.

$$\lim_{x \to \infty} \frac{x(x-1)}{x^2} = 1$$

Idea: b/c j is huge $\frac{j(j-1)}{j} \approx 1$

$$a^x = 1 + kx + \frac{k^2x^2}{2!} + \frac{k^3x^3}{3!} + \frac{k^4x^4}{4!}$$

e is the base such that this slope $k = 1$ ——————.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

Euler gave approx. to e by sub $x = 1$

$$e = e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{125} + \cdots$$

In about 1730, Euler gave

$e \approx 2.71828182845904523536028$

23 decimal places!

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

using similar methods: Euler derived

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

} These formulas are used in your calculator to compute (approximations) of $\sin(3)$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$