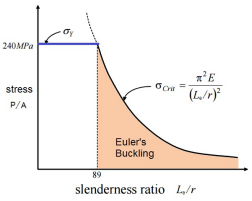
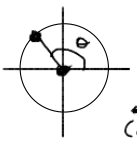


Leonhard Euler: 1707 - 1783

1. Switzerland, Russia, Berlin
2. math / phyics / astronomy / geography / engineer
   a. created graph theory & topology
   b. analytic number theory, complex analysis, calculus
   c. solidified the use of mathematical notation
      i. function notation: f(x)
      ii. greek letter: pi
      iii. imaginary number: i
      iv. summation: Sigma
      v. defined the constant e
      vi. introduced the use of exp function & logs in proofs
      vii. Euler's formula: exp(iz) = cos(z)+isin(z)
      viii. Pioneered analytic methods in number theory
      ix. hyperbolic trig functions
      x. continued fractions
   d. mechanics / fluid dynamics / optics / astronomy / music theory
      i. Integraded Leibniz's differential calculus with Newton's Fluxions
      ii. nature of & orbits of comets
      iii. foundations of longitude tables
      iv. Optics: foundations wave theory of light, (like Huygens)
      v. Structural engineering: Euler's critical load
      vi. Logic: Euler diagrams (before refinement to Venn diagrams)
3. Truly one of the greatest mathematicians in history.
   a. Laplace: "Read Euler, read Euler he is the master of us all.
   b. Gauss: "The study of Euler's works will remail the best school for the different fields of mathematics, and nothing else can replace it.
   c. Most prolific
      i. 850+ publications
      ii. 92 volumes
   d. Graph Theory / Topology: Seven Bridges of Konigsberg
   e. Basel Problem:
      i. What is the sum of reciprocals of squares?
      ii. Named after the town of Basel, Switzerland
         1. hometown to Euler & the Bernouli's
   f. Topology: Euler Characteristic

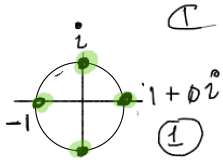
$(4,5)$

$(\cos\theta, \sin\theta)$ $\mathbb{R}^2$



(a,b)

$e^{i\theta} = \cos\theta + i\sin\theta$



$a + bi$

$2, 2^2, 2^3, 2^4, \ldots$

$1, i, i^2, i^3, i^4, i^5$

$(\sqrt{-1})(\sqrt{-1})$

$(-1)^{1/2}(-1)^{1/2} = -1$

$i^{-3} = i^2 \cdot i = -i$

$i^4 = i^3 i = (-i)(i) =$

$= -i^2$

$= -(-1) = 1$

Euler

$e^{i\pi} + 1 = 0$

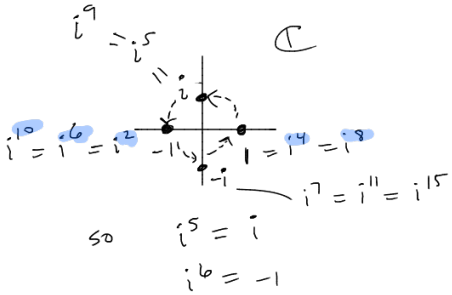
$e \approx 2.1718\ldots$, $i = \sqrt{-1}$

0, 1, π

Euler's Formula

The most beautiful equation in math: relates 4 of the most important numbers, e, i pi, 1 and 0

$$e^{i\pi} + 1 = 0$$



so $i^5 = i$

$i^6 = -1$

Start:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

set $x = i\theta$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!}$$

$$= 1 + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^6}{6!} + \dots \quad i\theta + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^7}{7!}$$

$$= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta^8}{8!} + \dots + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!}\right)$$

so $\boxed{e^{i\pi} + 1 = 0}$

$$\boxed{e^{i\theta} = \cos(\theta) + i \cdot \sin(\theta)}$$

so → plug in $\theta = \pi$ get $e^{i\pi} = \cos(\pi) + i \cdot \sin(\pi) = -1$

χ	Name of Solid	No. of Faces (F)	No. of Vertices (V)	No. of Edges (E)	Type of regular n-gon (polygon) at each face (n)	No. of faces at each vertex (k)	No. of degrees in each face angle	No. of deg. in ea. polyhedral angle
2	Tetrahedron	4	4	6				
2	Cube							
2	Octahedron							
2	Dodecahedron							
2	Icosahedron							

The Regular Solids[1]

Tetrahedron

Cube

Octrahedron

Dodecahedron

Icosahedron

Euler characteristic, $\chi(\text{polyhedron}) = V - E + F = 4 - 6 + 4 = 2$

"chi"

a number

Euler Characteristic is a topological invariant i.e., each of the Platonic solids a topological sphere

Exercise

$\chi(\text{triangle}) = 3 - 3 + 1 = 1$

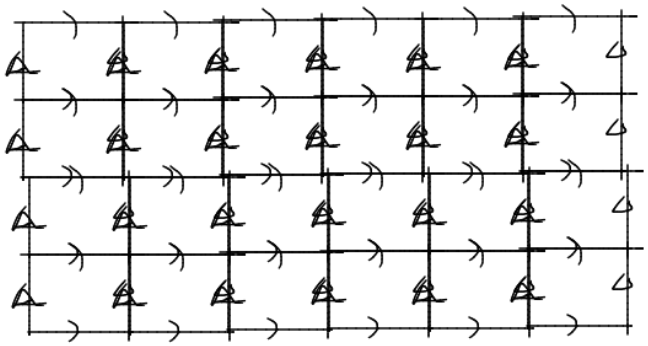
$\chi(\text{disk}) = 1 - 2 + 1 = 0$ $= 2 - 2(1)$

$\chi(\text{2 holes}) = 2 - 2g = 2 - 2(2) = -2$

genus, $g = 2$

Topology ≠ Geometry

torus



Euclidean