$$
\begin{array}{cccccccccccccccccc}
0 & & & & & & & & & & & & & & & & & \\
1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 8 & -8 & \ldots \\
\frac{1}{2} & -\frac{1}{2} & \frac{2}{2} & -\frac{2}{2} & \frac{3}{2} & -\frac{3}{2} & \frac{4}{2} & -\frac{4}{2} & \frac{5}{2} & -\frac{5}{2} & \frac{6}{2} & -\frac{6}{2} & \frac{7}{2} & -\frac{7}{2} & \frac{8}{2} & -\frac{8}{2} & \ldots \\
\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{3}{3} & -\frac{3}{3} & \frac{4}{4} & -\frac{4}{4} & \frac{5}{3} & -\frac{5}{3} & \frac{6}{3} & -\frac{6}{3} & \frac{7}{3} & -\frac{7}{3} & \frac{8}{3} & -\frac{8}{3} & \ldots \\
\frac{1}{4} & -\frac{1}{4} & \frac{2}{4} & -\frac{2}{4} & \frac{3}{4} & -\frac{3}{4} & \frac{4}{4} & -\frac{4}{4} & \frac{5}{4} & -\frac{5}{4} & \frac{6}{4} & -\frac{6}{4} & \frac{7}{4} & -\frac{7}{4} & \frac{8}{4} & -\frac{8}{4} &
\end{array}
$$

$$
\begin{array}{ccccccccccc}
 & 0 & 1 & -1 & 1/2 & 1/3 & -1/3 & & & & \\
\mathbb{Z} = \{ & 0, & 1, & -1, & 2, & -2, & 3, & -3, & 4, & -4, & 5, \ldots
\end{array}
$$

$$\overline{\mathbb{Q}} = \overline{\mathbb{Z}}$$

we'll see: $\underset{\aleph_0}{\overline{\mathbb{Z}}} < \underset{\aleph_1}{\overline{\mathbb{R}}} = \overline{\mathbb{R}^2} = \overline{\mathbb{R}^3} \ldots < \aleph_2 < \aleph_3 < \ldots$

extended the set of cardinal #'s

Aleph

Mon wk 14

Sizes of Sets:

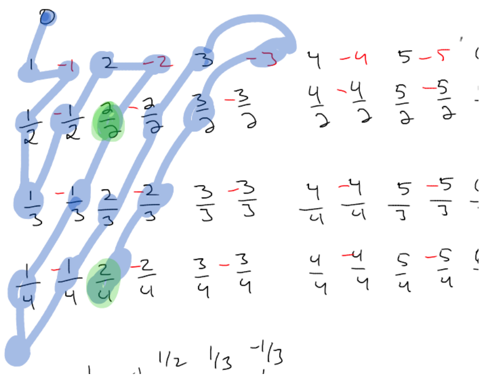
- finite, 1, 2, .. N
- countably infinite (countable): (bijection w/ counting #'s)

ex: $\mathbb{Z}$ = integers (countable)

$$\begin{array}{ccccccc} 0, & 1, & -1, & 2, & -2 & & \\ | & | & | & | & | & & \cdots \\ 0 & 1 & 2 & 3 & 4 & & \end{array}$$

$2\mathbb{Z}$ = even ints

$\mathbb{Q}$ = rationals



$$\begin{array}{ccccccccccc} 0 & 1, & -1, & \frac{1}{2}, & \frac{1}{3}, & -\frac{1}{2}, & 2, & -2, & -\frac{1}{3}, & \frac{1}{4} \\ | & | & | & | & | & | & | & | & | & | \\ 0 & 1 & 2, & 3, & 4, & 5, & 6, & 7, & 8, & 9, & 10 \end{array}$$

- uncountable: no bijection w/ $\mathbb{N}$

Are the reals #'s (0,1) countable ?

(If so, put them on a list

$.9999\ldots = .\overline{9}$

List of Reals

.4221999250

.5732199831

.221759832

.1245786531

.8473112333

.9238957122

.0307121725

.4688236941

.86753090000

.0000000001

.5 6 2 4 0 4 0 5 1 → this differs from each # above in @ least one spot

⇒ not on the list

choose # diff from digit 1 row 1

choose #, not digit 2 row 2

repeat

Diagonalization: Cantor's algorithm to produce a real # not on the list

Theorem: (0,1) is not countable

proof If so, write every # in (0,1) on list. Diagonalize to get some #, (real) that wasn't on the list.

Power Set

$S = \{\text{Jenna, Kenzie, Rose}\}$, $\overline{S} = 3$ (cardinality)

$P(S)$ = set of all subsets of S

$= \{\Phi, S, \{J,R\}, \{J,K\}, \{R,K\}, \{J\}, \{K\}, \{R\}\}$

$S = \{J,K,R\}$

Cardinality of $P(S) = 8 = 2^3$

Cantor asked, What's the Cardinality of $P((0,1))$

$P((0,1)) = \{\emptyset, \{Q, Q'\}, \{Q, Q', Q''\}, \{\frac{\sqrt{3}}{5}, \frac{\sqrt{2}}{2}\}\}$

Theorem: $P(0,1)$ is not contable. $\overline{P((0,1))} > \overline{(0,1)}$

proof: diagonalization

this leads to an infinite # of sizes of infinity

$$\overline{\mathbb{Z}} < \overline{(0,1)} < \overline{P(0,1)} < \overline{P(P(0,1))} < \overline{P(P(P(0,1)))} < \ldots$$

$$\aleph_0 \qquad \aleph_1 \qquad \aleph_2 \qquad \aleph_3 \qquad \aleph_4$$

aleph-naught