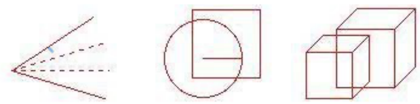
Hippocrtates' lune (born: 430 BC)



1. a merchant, was swindled and lost it all.

2. was one of the first paid teachers

3. Aristotle thought him foolish

4. In the 5th century, theorems piled up, requiring organization. He pioneered this, but his work was lost.

5. He was the pioneer of RAA, reductio ad absurdum - proof by contradition

6. One of the first to use letters to denote points on geometric figures.

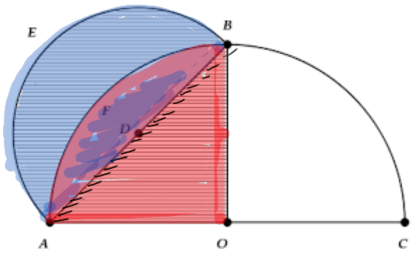
▼7. In Athens, 3 challenging problems had risen: <u>3 Great Problems of Antiquity</u>
   a. Square the circle
   b. Duplicate the cube
   c. Trisect any angle

▼8. His lune is quadrable & proof requires these simple ideas
   a. Thales Theorem
   b. Pythagorean Identity
   c. Ratio of areas of circles is ratio of squares of diameters * (akin to a formula for area of circle)
   d. AB = sqrt2*AO = sqrt2*/2 * AC
   e. AB^2 / AC^2 = 1/2
   f. Small semicircle = 1/2 large semicircle = AFBO quadrant
   g. Lune = triangle

Finish: Ch. 1

Next week: Ch. 2

Show Hippocrates' Lune is Quadrable



Show $|\text{Lune } AEB| = |\triangle AOB|$

Tool: Ratio of areas of two circles equals

Ratio of square of diameters

$$\boxed{\begin{array}{l} \dfrac{\pi r_1^2}{\pi r_2^2} = \dfrac{r_1^2}{r_2^2} = \dfrac{D_1^2}{D_2^2} \\[2ex] D = 2r_1 \\[2ex] \dfrac{D_1^2}{D_2^2} = \dfrac{4r_1^2}{4r_2^2} = \dfrac{r_1^2}{r_2^2} \end{array}}$$

Strategy:

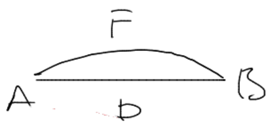
$|\text{⌓} AEBD| = |\text{◿} AOBF|$

$$\text{Ratio of } \frac{\text{Large Semi Circle}}{\text{Small Semi Circle}} = \frac{AC^2}{AB^2} = \frac{(2AO)^2}{AB^2} = \frac{4AO^2}{AB^2}$$

Pythag. th. : $AB^2 = AO^2 + OB^2 = 2AO^2$ ... together $\boxed{\text{Ratio} = 2}$

$$\text{Blue} = \frac{\text{Large Semi Circle}}{2} = \text{Red}$$

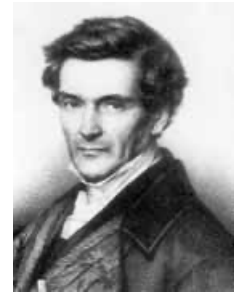
Since



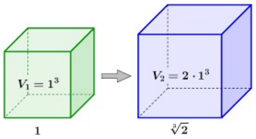
is common to both, we're done.

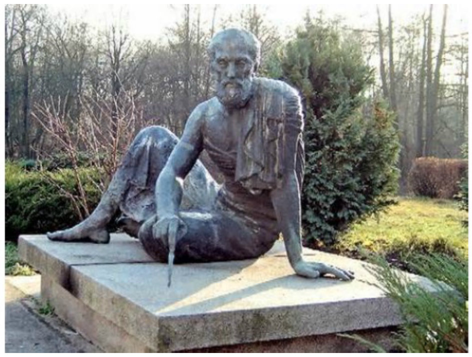
Duplication of the Cube

1. Great problem of antinquity
2. Hippocrates found a new approach - equating the problem to the constructibility of a number - the square root of 2.
3. Roughly 2000 years later, this was shown to be impossible.
   a. **Pierre Wantzel (1837)**
   b. https://www.ams.org/journals/bull/1918-24-07/S0002-9904-1918-03088-7/S0002-9904-1918-03088-7.pdf
   c. Wantzel's work initially went unnoticed, mentioned once in **1871**, and was finally picked up around **1920** !
      i. In Gauss's words, such results were of interest only in so far as they would prevent mathematicians from wasting their time trying to do impossible mathematics.
         1. **Article on Why Wantzel was overlooked:** https://www.sciencedirect.com/science/article/pii/S031508600900010X
      ii. **The dominant paradigm in Wantzel's day: constructive & quantitative**
      iii. **Eventually, well known mathematician's Abel & Cauchy pushed the paradigm into more qualitative & conceptual areas**
      iv. By then, Wantzel's name had been forgotten.
   d. In this work he also proved the impossibility of trisecting an arbitrary angle.
   e. With Lindemann's work in 1877, the circle could not be squared - but the way it seemed to be framed was: pi is transcendental - this is different from what Wantzel did (he showed a cubic polynomial is irreducible over Q.



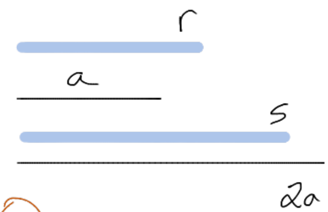
Duplicating the Cube





Hippocrates (400 BC) reduced the problem to

$\boxed{\text{given lengths a, 2a construct proportionals r, s such that a/r = r/s = s / 2a}}$



$$\boxed{\frac{a}{r} = \frac{r}{s} = \frac{s}{2a}}$$

go from here to $\sqrt[3]{2}$

① cross-multiply

$$\boxed{r^2 = as}$$

② cross-mult First / Last

$$\boxed{2a^2 = rs}$$

$s^2 = \frac{4a^4}{r^2}$

isolate s & square ③

④ cross mult 2nd last

$$\boxed{2ar = s^2}$$

$$\cancel{2}ar = \frac{\cancel{4}^{2}a^{\cancel{4}\,3}}{r^2}$$

⑤ cancel

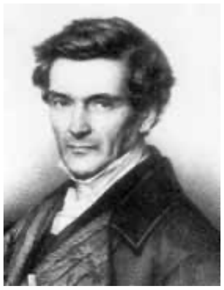
$$r^3 = 2a^3$$

$$\boxed{r = \sqrt[3]{2}a}$$

This means we need to construct a new segment that is cube root of 2 as long as segment a.

This turns out to be impossible w/ compass & straightedge.

Pierre Wantzel:
France - 1830



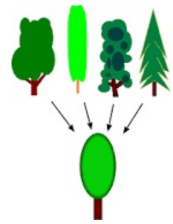
"Wantzel." Pinet says of him: "Endowed with extreme vivacity of impressions and with truly universal aptitudes, he carried off the prize for a French dissertation and a Latin dissertation at a general competition and, the year following (1832), entered with first rank the Polytechnic School—a double success before unheard of. He studied with inveterate zeal German and Scotch philosophers; he threw himself into mathematics, philosophy, history, music, and into controversy, exhibiting everywhere equal superiority of mind." He became élève-ingénieur des Ponts et Chaussées, then ingénieur; he was appointed répétiteur about the time when such positions were held by Comte, Transon, Bertrand, Bonnet, Catalan, Leverrier, and Delaunay. In 1843 he became examinateur d'admission. Saint-Venant says of him: "He was blameworthy for having been too rebellious to the counsels of prudence and of friendship. Ordinarily he worked evenings, not lying down until late; then he read, and took only a few hours of troubled sleep, making alternately wrong use of coffee and opium, and taking his meals at irregular hours until he was married. He put unlimited trust in his constitution, very strong by nature, which he taunted at pleasure by all sorts of abuse. He brought sadness to those who mourn his premature death."

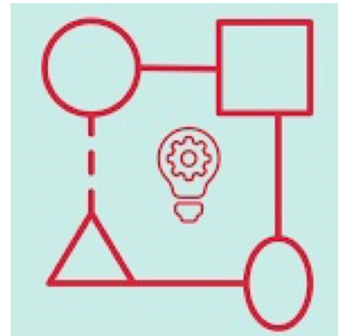
The Royal Society Catalogue of Scientific Papers quotes the titles of 18 papers by Wantzel, and of three more which he brought out jointly with Saint-Venant.

Read More

1. https://www.ams.org/journals/bull/1918-24-07/S0002-9904-1918-03088-7/S0002-9904-1918-03088-7.pdf

Wantzel's work **1830** initially went unnoticed, mentioned once in **1871**, and was finally picked up around **1930** !





Read More

1. **Article on Why Wantzel was overlooked:** https://www.sciencedirect.com/science/article/pii/S031508600900010X