MA 484 wk 3 Fri

Homework due <u>Wed</u> Feb 5

Kinds of Numbers
Classifications

- Integers : $\mathbb{Z}$ (zahlen, German word for 'number')
- odd / even
- algebraic / transcendental — π (Lindemann) 1800's

both partition $\mathbb{R}$ — real #'s

algebraic: solutions to a polynomial $\sum_{i=0}^{m} a_i x^i$ | $m > 0$

transcendental: not

History: constructible #'s / not-constructible

$\sqrt{2}$ — $\sqrt[3]{2}$

Algebraic Numbers

"there exists"

247 is algebraic b/c ∃ a poly w/ int coeffs "that kills" 247

plus 247 in, get 0

(1) $f(x) = x - 247$ kills 247

$\Rightarrow$ Generalizing every integer is algebraic

(2) $\frac{7}{3}$ is algebraic b/c

$P(x) = 3x - 7$

$x = \frac{7}{3}$

$x - \frac{7}{3} = 0$

$3x - 7 = 0$

Note: $f(x) = x - \frac{7}{3}$ isn't a poly w/ int. coefs

$\Rightarrow$ doesn't work

$\Rightarrow$ Generalizing $\mathbb{Q} \subset$ Algebraic

(rational #'s (fractions))

(3) $\phi = \frac{1+\sqrt{5}}{2}$ the golden ratio, is algebraic

$\boxed{P(x) = x^2 - x - 1}$

$x = \frac{1+\sqrt{5}}{2}$

$2x = 1 + \sqrt{5}$

$2x - 1 = \sqrt{5}$ isolate radical

$4x^2 - 4x + 1 = 5$

$4x^2 - 4x - 4 = 0$

$x^2 - x - 1 = 0$ when ever

(4) $\sqrt[3]{5}+\sqrt{7}$ is algebraic

cube a Binomial

Pascal's Trangle

$$\begin{array}{c} 1 \\ 1\quad 2\quad 1 \\ 1\quad 3\quad 3\quad 1 \\ 1\quad 4\quad 6\quad 4\quad 1 \\ 1\quad 5\quad 10\quad 10\quad 5\quad 1 \\ 1\quad 6\quad 15\quad 20\quad 15\quad 6\quad 1 \end{array}$$

Degree Sum is 3

$(a-b)^3$ ⇒ signs alternate

set (1) $x=\sqrt[3]{5}+\sqrt{7}$

(2) $x-\sqrt{7}=\sqrt[3]{5}$

↓ cube

(3) $(x-\sqrt{7})^3=(\sqrt[3]{5})^3=5$

$$x^3-3x^2\sqrt{7}+3x\cdot 7-7\sqrt{7}=5$$

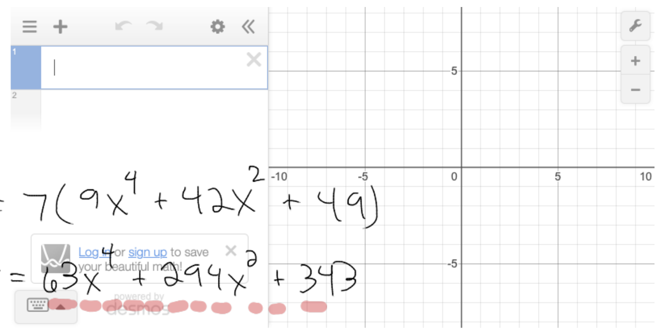
(4) get radicals isolated

$$x^3+21x-5=\sqrt{7}(3x^2+7)$$

(5) square:

$$(x^3+21x-5)^2=(\sqrt{7}(3x^2+7))^2=7(9x^4+42x^2+49)$$

$$x^3(x^3+21x-5)+21x(x^3+21x-5)-5(x^3+21x-5)=63x^4+294x^2+343$$



$$x^9+21x^4-5x^3+441x^2-105x+25$$
$$+21x^4-5x^3 \qquad -105x$$

$$x^9+42x^4-10x^3+441x^2-210x+25$$

$$\boxed{x^6-21x^4-10x^3+147x^2-210x-318}$$

$\dfrac{1}{\sqrt[3]{5}+\sqrt{7}}$ is alg.

$x = \dfrac{1}{\sqrt[3]{5}+\sqrt{7}} \xrightarrow[\text{mult}]{\text{cross}} \sqrt[3]{5}\cdot x + \sqrt{7}\,x = 1$ repeat ...

#10/

$f(x) = \cancel{x}^{2} - 7\cancel{x} + 12 = (x-3)(x-4)$

$f(3) = 0$

$f(4) = 0$

$g(x) = 12x^2 - 7x + 1$

$g\left(\frac{1}{3}\right) = 12\left(\frac{1}{3}\right)^2 - 7\left(\frac{1}{3}\right) + 1$

$= \frac{12}{9} - \frac{7}{3} + 1 = \frac{12}{9} - \frac{21}{9} + \frac{9}{9} = \frac{0}{9} = 0$