

Euclid's Proof of the Pythagorean Theorem

- ▼ 1. Greece flourished in the 150 years between Hippocrates & Euclid.
 - ▼ a. Plato (427 BC) and his student Eudoxus (408 BC) led the way
 - i. Many sophists (clever know-it-alls who taught for money) wandered the land - some brilliant, some quacks
 - ii. **Socrates'** disciple was **Plato**, and his student was **Aristotle**
 - ▼ b. Plato traveled until age 40, then founded the Academy in Athens. The intellectual center of the world, headquarters of Greek wisdom.
 - i. Learned from the Pythagoreans in Italy, visited Egypt, etc. Sold as a slave, but then ransomed by his friends.
 - ii. His school is the spiritual ancestor of our institution of higher learning, the land had belonged to the hero Akademos.
 - iii. As was custom, to establish legitimacy the Academy was a "religious brotherhood" worshiping the Muses
 - iv. For 900 years it stood as intellectual center of Greece until 529 AD, was closed b/c it was "pagan" and of "perverse learning"
 - ▼ c. Math was king, Geometry was required for entrance.
 - i. Plato disliked "applied mathematics" and geometric instruments, embracing only the straightedge and compass
 - ▼ d. **Eudoxus** was poor, commuted in to Athens, and had two main pieces of mathematics, was 2nd only to Archimedes
 - ▼ i. Theory of proportion
 - 1. Many proofs of "similar triangles have equal ratios of corresponding sides" required the (false) commensurability of numbers.
 - 2. Eudoxus' devised a proof that did not appeal to commensurability.
 - 3. His proof is found in Book V of the Elements (Euclid didn't prove everything there.)
 - ▼ ii. Method of Exhaustion
 - 1. Main idea of limits/calculus.
 - 2. Archimedes used/credited Eudoxus' idea to find the area of a circle.
 - ▼ e. Alexander the Great & Alexandria
 - i. 332 BC Alex (20 y/o from Macedonia) conquered Egypt, established a new city Alexandria
 - ii. Alexandria grew while Alex "conquered the world" and became the intellectual center
 - iii. 500 K people and 600K (really?) Papyrus rods, Alexandrian Library & Museum > Academy,
 - iv. The Museum (the oldest university) was heaven on earth for thinkers, food, space community.
 - v. Alex conquered Greece, Egypt, most of Asia Minor, and died trying to capture India.
 - vi. Alexandria remained central & powerful for 300 years - the Hellenistic (Greek-like) age, until the Roman Empire.
 - vii. 300 - 100 BC is the 2nd most mathematically productive era ever. #1 = Kepler - Gauss (1600 -1850)
 - viii. Archimedes, Eratosthenes, Apollonius Pappus, Claudius Ptolemy, Diophantus & Euclid all worked here at some time.

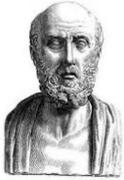
wk 3'

wed: Your favorite proof of the Pythagorean theorem
next (wed) written HW.

Today: Euclid's Proof of P_y-Th.

- History

- Proof



Hippocrates - 430 BCE



Sophist

Greece flourished during this period



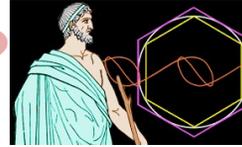
Euclid - 300 BCE



Socrates



Pythagoras



Eudoxus - 408 BCE

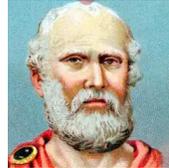


A Diagram and a step,
not a diagram $\frac{1}{4}$ a coin.



Spiritual ancestor
of the University

Academy



Plato - 427 BCE

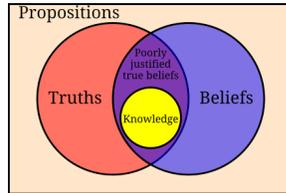


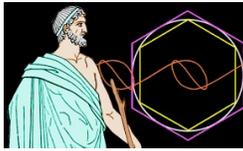
Aristotle

In ancient Greece,
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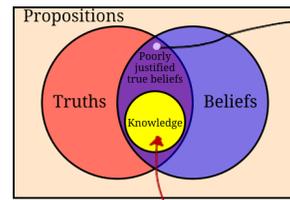
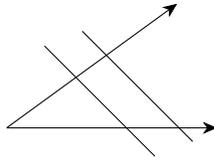
lasted 900
years, until
529 AD

closed due to: pagan, perverse learning





Eudoxus - 408 BCE



Later, it moved here

Initially (before "similar Pythagoras" theorem)

His proof that "similar triangles have equal ratios of corresponding sides" solved the logical scandal of ancient Greece.

Euclid's

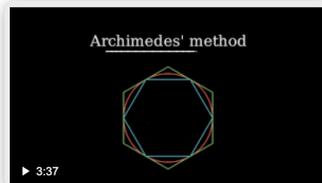
This proof is found in Book V of the Elements.

Eudoxus's proof didn't need this 'faulty' assumption

* Euclid didn't create all proofs contained, he organized / re-wrote many.

- proofs of it relied upon 'commensurate lengths'

- Pythagoreans $\Rightarrow \sqrt{2} \neq \frac{a}{b}$
 \Rightarrow Commensurate Length = FALSE!
 Assumption



YouTube
 Archimedes' method/method of exhaustion ...

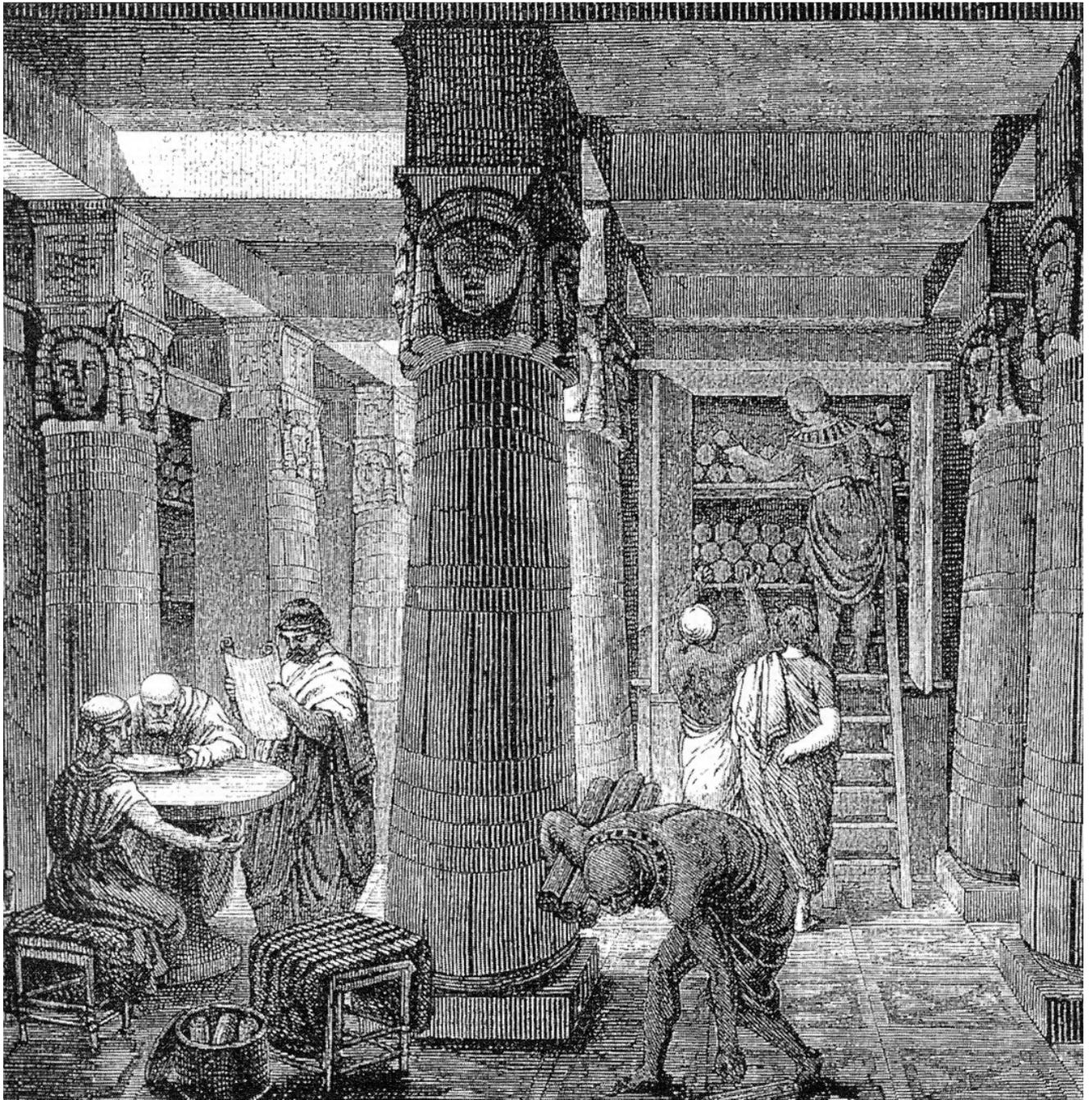
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- 332 BC Alexander the Great
- Alexandria = intellectual center of Egypt



500K people in Alexandria
~ 600K Papyrus Rods.

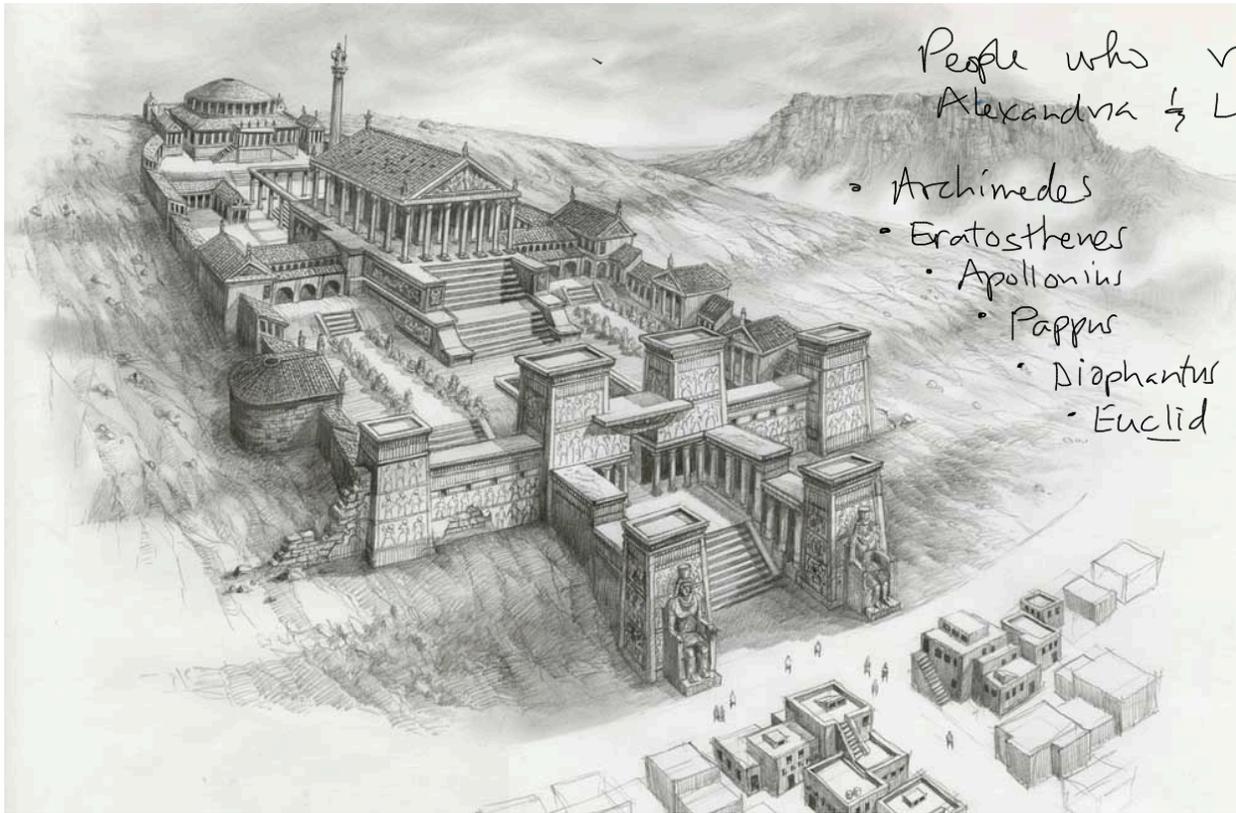


19th century depiction of Library @ Alexandria

Euclid's Proof of the Pythagorean Theorem

- ▼ 1. Euclid came to Alexandria to establish a school of mathematics (300 BC)
 - a. Trained at the Academy, but hereafter Greek mathematics had Alexandrian roots
 - b. Founded a school in Alexandria
- ▼ c. Two Euclidean stories
 - ▼ i. King Ptolemy asked about a shorter way to learn geometry than reading The Elements
 - 1. "There is no royal road to geometry"
 - ▼ ii. A young student asks Euclid: "But what shall I get by learning these things?"
 - 1. Euclid points out that knowledge is useful for its own sake then ...
 - ▼ 2. Says to servant, "Give this man a coin, since he must make a profit from what he learns"
 - a. This might come from the Pythagorean slogan, "A diagram and a step (in knowledge), not a diagram and a coin."
- ▼ d. The Elements of Geometry - only the Bible has been studied more
 - i. "The most splendid creation of the Greek mind" - Burton
 - ii. 13 books, 435 propositions, 2000+ editions, essential to a liberal education
 - iii. plane/solid geometry & number theory
 - ▼ iv. He organized geometric knowledge beautifully, clearly, all stemming from a few basic assumptions - axiomatically
 - 1. 5 geometric postulates, 5 common notions, 23 definitions = 435 propositions
 - 2. the choice of axioms, the arrangement of the propositions, the rigor of demonstration are his own and is amazing
 - 3. a minimum of assumptions and very little that is superfluous
 - v. No circular reasoning - required axioms (assumed without proof)
 - vi. When Rome fell, Arab scholars carried it to Baghdad, reappeared in the Renaissance,
 - vii. Studied by Newton & Leibniz, Napoleon, Lincoln (40 y/o read to train his logical approach, Bertrand Russell)

The Library @ Alexandria Euclid:



People who visited
Alexandria & Library

- Archimedes
- Eratosthenes
- Apollonius
- Pappus
- Diophantus
- Euclid