MA484 - Week 4 - Wednesday

Homework discussion. Make sure you have thought about the problems before reading the solutions that follow. It won't sink in otherwise.

#10/ If c is algebraic then $1/c$ is algebraic too $(c \neq 0)$

since #7 relates, let's examine it. ✓

$\Rightarrow$ c is zero of poly. $P(x)$

$\Rightarrow$ There is some $P(x) = \sum_{k=0}^{n} a_k x^k$ for which

$$P(c) = \sum_{k=0}^{n} a_k c^k = 0$$

eg. $c = 2$.

$P(x) = x^2 - 5x + 6$

$P(2) = 2^2 - 5(2) + 6 = 0$

Q: Can I build new poly: $Q(x)$ from $P(x)$ s.t. $Q(1/2) = 0$.

$$\left(\frac{1}{2^2}\right)\left(2^2 - 5(2) + 6\right) = (0)\left(\frac{1}{2^2}\right) = \left(\frac{1}{2}\right)^{2}$$

because I can & it works

degree

$$0 = \frac{2^2}{2^2} - \frac{5 \cdot 2}{2^2} + \frac{6}{2^2} = 1 - 5\left(\tfrac{1}{2}\right) + 6\left(\tfrac{1}{2^2}\right)$$

$$0 = 1 - 5\left(\tfrac{1}{2}\right) + 6\left(\tfrac{1}{2}\right)^2$$

this eq'n suggests the answer:

$Q(x) = 1 - 5(x) + 6(x)^2$ or $\boxed{6x^2 - 5x + 1 = Q(x)}$

In general gory detail:

assumption of c being algebraic implies:

$$(5x) \lim_{h \to 0} f(h) = \lim_{h \to 0} 5x \cdot f(h)$$

$$P(c) = \sum_{k=0}^{\overset{\text{degree}}{n}} a_k c^k = 0$$

k varies, n is constant

mult. by $(c^{-1})^n$

$$\left(\frac{1}{c}\right)^n = (c^{-1})^n = c^{-n}$$

$$c^{-n} \sum_{k=0}^{n} a_k c^k = 0 \cdot c^{-n} = 0$$

$$\sum_{k=0}^{n} a_k c^k \cdot c^{-n} = \sum_{k=0}^{n} a_k c^{k-n}$$

$$= \sum_{k=0}^{n} a_k c^{-1(n-k)}$$

$$c^{-1(n-k)} =$$

$$= \sum_{k=0}^{n} a_k (c^{-1})^{n-k}$$

$$c^{a \cdot b} = (c^a)^b$$

suggests what the polynomial should be that kills $\frac{1}{c}$

$$= \sum_{k=0}^{n} a_k \left(\frac{1}{c}\right)^{n-k}$$

Transcendental #'s : Def'n : not algebraic

: not the solution to

$$\sum_{k=0}^{n} a_k x^n = 0$$

$$a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

$a_i \in \mathbb{Z}$.
integers

#12

Suppose $\pi + 1$ is algebraic.

In #11 we showed c being algebraic implied $c+1$ is.
↳ this technique can be used to show $c + r$ is algebraic for any $r \in \mathbb{Z}$.

By #11 π must be algebraic since
π is an additive integer diff than $(\pi + 1)$.

#11 c is algebraic.
$1+c$ is too

Ex $f(x) = x^2 - 2x + 1$ $(1,0)$

$g(x) = f(x-4) = (x-4)^2 - 2(x-4) + 1$ $(5,0)$

$= x^2 - 8x + 16 - 2x + 8 + 1$

$g(x) = x^2 - 10x + 25$

$g(5) = 25 - 50 + 25 = 0$

Show

$13 + c$ is algebraic

#7/ Show $x = \sqrt[3]{3} + \sqrt{2}$ is algebraic.

what does this mean? ($\sqrt[3]{2} + \sqrt{2}$ is a zero of some polynomi(with integer coeffs)

(idea: try an easier one)

$x = \sqrt{2}$ is algebraic

$x^2 = 2$

$x^2 - 2 = 0$

similarly:

$(x - \sqrt{2}) = \sqrt[3]{3}$

$(x - \sqrt{2})^3 = 3$

(expand.

$x^3 - 3x^2 \cdot \sqrt{2} + 3x \cdot 2 - 2^{3/2} = 3$

$(\sqrt{2})^3 = (2^{1/2})^3$

(I)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

stop @ 3

coeffs.

(II) degree sum of each term is 3

(III) signs alternate.

$(a-b)^3 = a^3 - a^2 b + ab^2 - b^3$

$x^3 - 3x^2 \cdot \sqrt{2} + 3x \cdot 2 - 2^{3/2} =$ $= 3$

separate left/right into integer / non-ints:

$x^3 + 6x - 3 = 3\sqrt{2}x^2 + 2\sqrt{2} = \sqrt{2}(3x^2 + 2)$

$x^3 + 6x - 3 = \sqrt{2} \cdot (3x^2 + 2)$

now square both sides

$(x^3 - 6x - 3)^2 = 2(3x^2 + 2)^2$

you can stop here, stating from this we can expand getting a polynomial in standard form

(b) $x = \dfrac{1}{\sqrt[3]{3} + \sqrt{2}}$

$x(\sqrt[3]{3} + \sqrt{2}) = 1$

distrubute & expand..

(#10

$\sqrt[3]{3} \cdot x + \sqrt{2}x = 1$

$\sqrt[3]{3}x = 1 - \sqrt{2}x$

$3x^3 = (1 - \sqrt{2}x)^3$ similar to before.