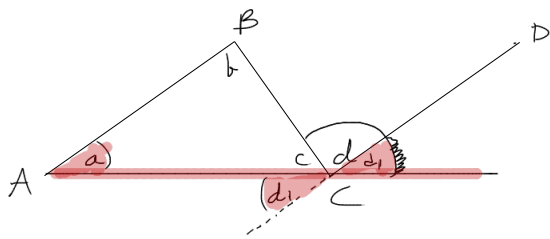


Euclid's proof of AAS

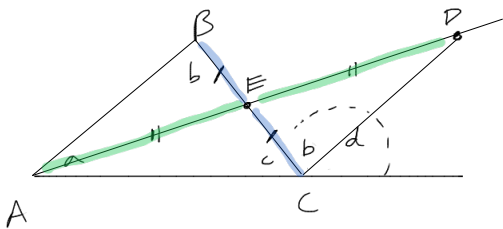
Exterior Angle theorem



angle d is exterior to the $\triangle ABC$

$d > a$ & $d > b$

Proof (I) extend line from c , \parallel to \overline{AB} , get $\overline{CD} \parallel \overline{AB}$ & $d_1 = a$ since $\parallel \Rightarrow \text{Alt Angs} =$
 note d_1 is contained inside d .
 $d > d_1 = a$ or Eucl. Geom $\Rightarrow a + b + c = \text{circumference} = a + d_1 + (d - d_1) = a + d$
 $b + c = d \Rightarrow d > b$

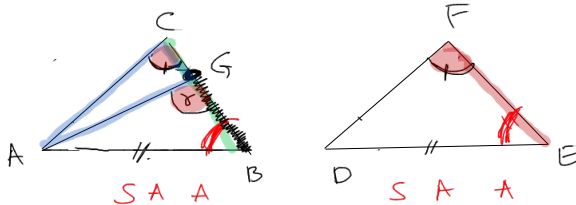


1. Bisect BC , extend to D s.t. $AE = ED$
2. Form CD
- 3 d contains angle $b \Rightarrow d > b$

AAS

(triangle congruence theorem)

Assumptions: $\triangle ABC, \triangle DEF$



$\angle C = \angle F$

$\angle B = \angle E$

$\overline{AB} = \overline{DE}$

Show $\triangle ABC \cong \triangle DEF$

Proof If $\overline{CB} = \overline{FE}$ done by SAS, so assume they're not congruent, assume $\overline{CB} > \overline{FE}$.
 So there must be some pt G on BC s.t. $\overline{BG} = \overline{FE}$. Now consider $\triangle ABG, \triangle ABG \cong \triangle DEF$ by SAS. Thus $\angle G = \angle F$.
 View $\angle G$ as an exterior angle to $\triangle AGC$, i.e., $\angle G > \angle C \Rightarrow \angle G > \angle F$ contradiction!
 $\Rightarrow \triangle ABC \cong \triangle DEF. !$

Euclid's Elements - Book I

▼ 1. Propositions 1 - 26

▼ a. Triangle Congruence

i. SAS

▼ 1. Uses superposition - moving one object onto another to define congruence.

▼ a. What is superposition?

i. group theory - Charles Dodgson (Lewis Carroll) would say no!

2. Modern development: SAS is an axiom

ii. SSS & ASA follow from SAS

▼ b. Constructing perpendiculars

i. It is implied that exactly ONE perp can be constructed

▼ c. Isosceles triangles

i. Base angles are equal

ii. The last proposition of Euclid taught in the Middle Ages

▼ iii. Flight of the fools

1. Students abandoned geometry afterward

▼ iv. Bridge of Asses - pons asinorum

1. The proof's figure resembled a bridge, and stumped many

2. Euclid's proof was complicated, Proclus' shorter, Pappus' was flip it "Irish Bull"

▼ 2. Propositions 27+ : Parallels

▼ a. AIA congruent implies parallel

i. doesn't use V

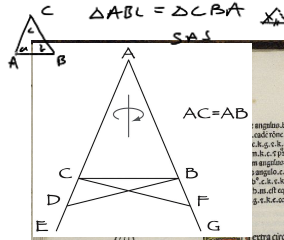
▼ b. Parallel lines CAN be constructed without V

▼ c. Sum of angles is 2 right angles (180 degrees)

i. requires V.

▼ d. Many pieces of geometry rely on V.

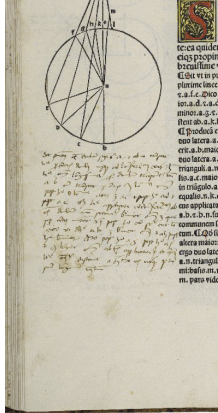
i. Quadrilaterals



LIETR

intra circulum puncto fixato ab eo ad circumferentiam lineae plures, ducuntur circulum fecundo, que sipe continentur in altero eorum et in longiora. Et centro quodam propositioe recte remouebatur loquor. Et nec vero partiales ad circulem extrinsecum applicat esse propositioe remouebatur bis in circulo. Quae vero que lineae bis in circulo sunt esse propositioe equalia sunt.

Propositio 8.



Et si in puncto a quo intra circulum sita sunt plures lineae ad circulem extrinsecum, non sunt equalia. Quae vero que lineae bis in circulo sunt esse propositioe equalia sunt.

Propositio 9.

Propositio 10.

Propositio 11.

Propositio 12.

Propositio 13.

Propositio 14.

Propositio 15.

Propositio 16.

Propositio 17.

Propositio 18.

Propositio 19.

Propositio 20.

Propositio 21.

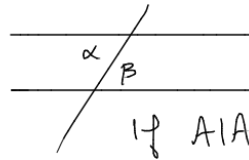
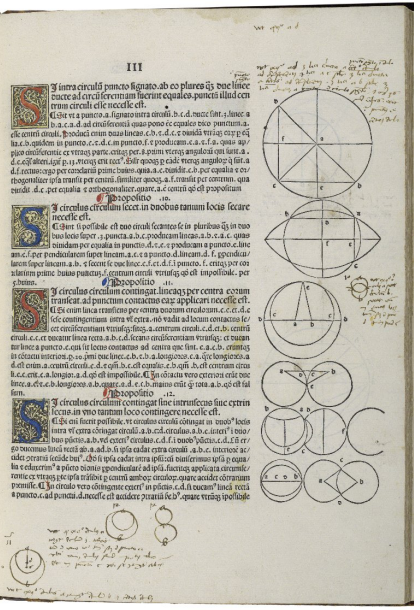
Propositio 22.

Propositio 23.

Propositio 24.

Propositio 25.

Propositio 26.



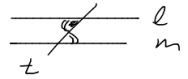
if AIA's are same, then \parallel



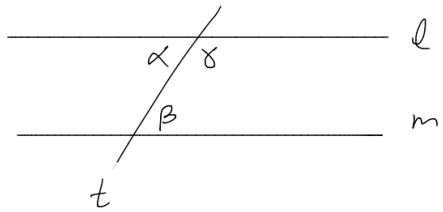
$\alpha + \beta + \gamma = 180^\circ$
uses V.

} Euclidean Geometry.

AIA theorems



① If $l \parallel m$ & they're intersected by $t \Rightarrow$ AIA's are equal



Assume: $l \parallel m$. Show $\alpha = \beta$.

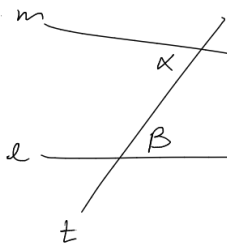
If not ($\alpha \neq \beta$) then assume $\alpha > \beta$.

since $\alpha + \gamma = 180^\circ$ (2RA's)

$\beta + \gamma < 180^\circ$ (2RA's)

\therefore Euclid's \Rightarrow l must meet m , \otimes

② If AIA's are = then the lines are \parallel

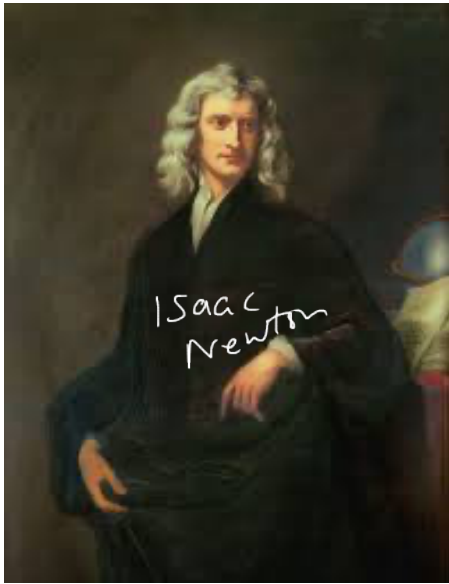


Assume: $\alpha = \beta$, & the lines intersect @ C.

this forms a triangle of which α is an exterior angle

Ext. Angle $\Rightarrow \alpha > \beta$.

\otimes



Isaac
Newton



Leibniz



Lincoln

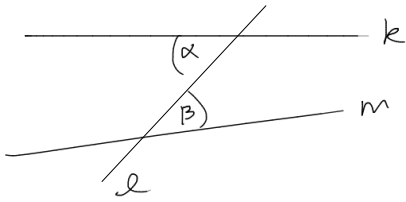


Napoleon



Bertrand Russell

ALTERNATE INTERIOR ANGLES

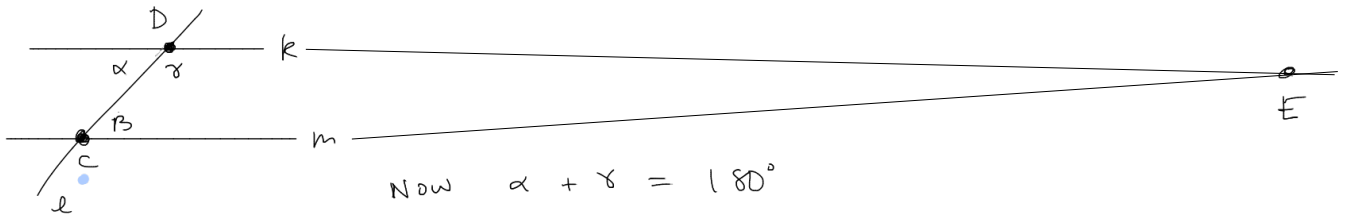


lines k, m & transversal l make alt. int. angles AIA's

Prop 1: If AIA's are equal then the two lines are parallel,

proof:

Assume angle $\alpha = \beta$, l is transversal on m & k .
Also assume k and m intersect, @ E



Now $\alpha + \gamma = 180^\circ$
 $\therefore \beta + \gamma = 180^\circ$

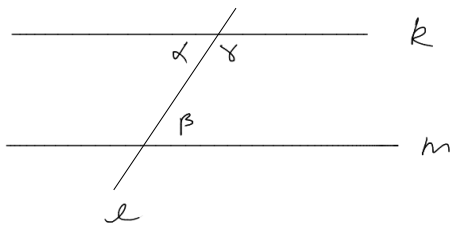
Now $\triangle DCE$ has an exterior angle α . By the Ext. Angle Thm $\alpha > \beta$ (X)

$\therefore E$ cannot exist. $\Rightarrow k \parallel m$

(Note, the proof above did NOT use the parallel postulate, and is true in NEUTRAL geometry (both))

Prop (2.9) If two lines k, m are parallel then their AIA's are equal,

proof.



If $\alpha \neq \beta$ then $\alpha > \beta$

$\alpha + \gamma = 180^\circ$

$\beta + \gamma < 180^\circ$

\Rightarrow Euclid's I.V. \Rightarrow

same side int angle add to less than 180°

\Rightarrow lines intersect (X)

$\alpha + \gamma = 180$

As

Conclusion: $\alpha = \beta$