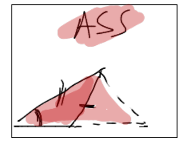
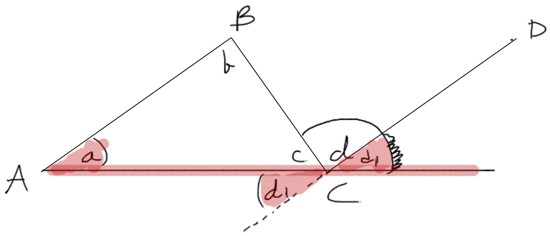
Euclid's proof of AAS

SAS
ASA
AAS



Exterior Angle theorem



angle d is "exterior to the ΔABC

$d > a$ & $d > b$

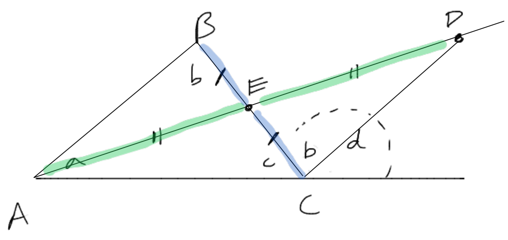
Proof

(I) extend line from C, $\parallel$ to $\overline{AB}$, get $\overline{CD} \parallel \overline{AB}$ & $d_1 = a$ since Euclid $\parallel \Rightarrow$ AIA =

note d_1 is contained inside d.

$d > d_1 = a$ ■ or Eucl. Geom $\Rightarrow$ $a + b + c = 2RA = a + d_1 + (d - d_1) = a + d$

$b + c = d \Rightarrow$ $c \neq 0$: $d > b$

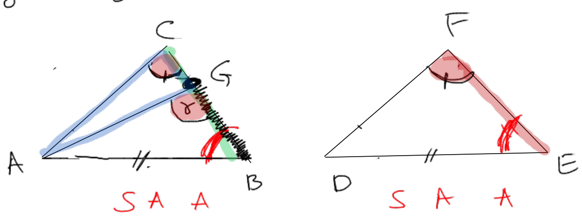


1. Bisect BC, extend to D s.t.
   AE = ED
2. Form CD
3. d contains angle b $\Rightarrow$

$d > b$

AAS

(triangle congruence theorem



Assumptions: ΔABC, ΔDEF

$\angle C = \angle F$

$\angle B = \angle E$

$\overline{AB} = \overline{DE}$

Show $\Delta ABC \cong \Delta DEF$

Proof If $\overline{CB} = \overline{FE}$ done by SAS, so assume they're not congruent, assume $\overline{CB} > \overline{FE}$.

So there must be some pt G on BC s.t. $\overline{BG} = \overline{EF}$. Now consider

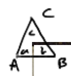
ΔABG, $\Delta ABG \cong \Delta DEF$ by SAS. thus $\gamma = \angle F$. contradiction!

View γ as an exterior angle to ΔAGC, i.e., $\gamma > \angle C \Rightarrow \gamma > F$

$\Rightarrow$ $\Delta ABC \cong \Delta DEF$. !

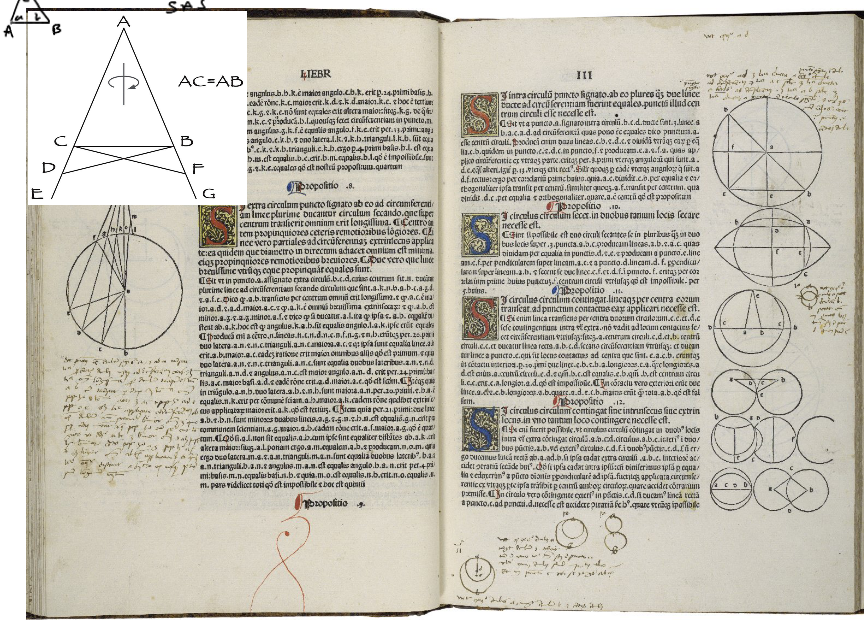
Euclid's Elements - Book I

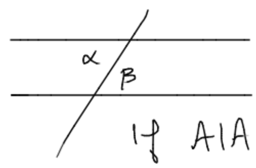
1. **Propositions 1 - 26**
    - a. **Triangle Congruence**
        - i. SAS
            - 1. Uses superposition - moving one object onto another to define congruence.
                - a. What is superposition?
                    - i. group theory - Charles Dodgson (Lewis Carrol) would say no!
            - 2. Modern development: SAS is an axiom
        - ii. SSS & ASA follow from SAS
    - b. **Constructing perpendiculars**
        - i. It is implied that exactly ONE perp can be constructed
    - c. **Isosceles triangles**
        - i. Base angles are equal
        - ii. The last proposition of Euclid taught in the Middle Ages
        - iii. Flight of the fools
            - 1. Students abandoned geometry afterward
        - iv. Bridge of Asses - pons asinorum
            - 1. The proof's figure esembled a bridge, and stumped many
            - 2. Euclid's proof was complicated, Proclus' shorter, Pappus' was flip it "Irish Bull"
2. **Propositions 27+ : Parallels**
    - a. **AIA congruent implies parallel**
        - i. doesn't use V
    - b. **Parallel lines CAN be constructed without V**
    - c. **Sum of angles is 2 right angles (180 degrees)**
        - i. requires V.
    - d. **Many pieces of geometry rely on V.**
        - i. Quadrilaterals



$\triangle ABC = \triangle CBA$

SAS

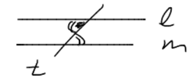




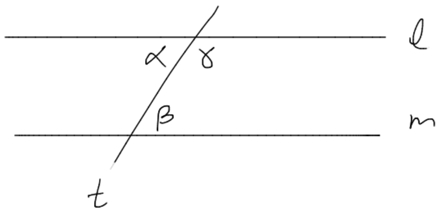
If AIA's are same, then ||

$\left.\begin{array}{l}\alpha + \beta + \gamma = 180^\circ \\ \text{uses V.}\end{array}\right\}$ Euclidean Geometry.

AIA theorems



(1) If $l \parallel m$ & they're intersected by t $\Rightarrow$ AIA's are equal



Assume: $l \parallel m$. Show $\alpha = \beta$.

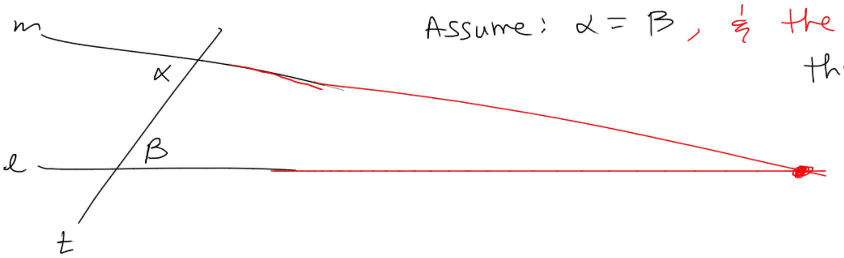
If not ($\alpha \neq \beta$) then assume $\alpha > \beta$.

since $\alpha + \gamma = 180^\circ$ (2 RA's)

$\beta + \gamma < 180^\circ$ (2 RA's)

By Euclid's V $\Rightarrow$ l must meet m, X

(2) If AIA's are = then the lines are $\parallel$

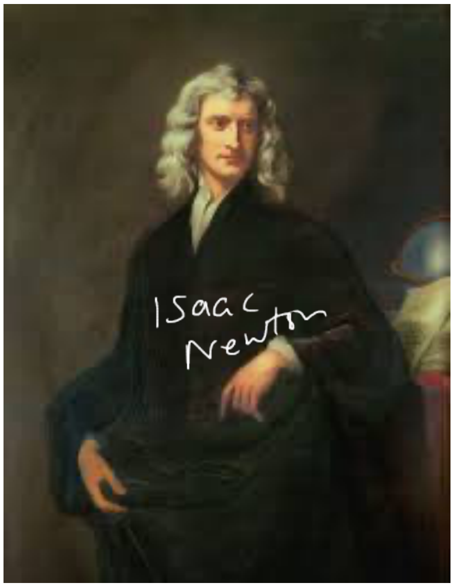


Assume: $\alpha = \beta$, & the lines intersect @ C.

this forms a triangle of which α is an exterior angle

Ext. Angle $\Rightarrow$ $\alpha > \beta$.

X

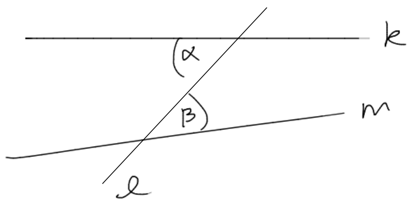


Lincoln

Napolean

Bertrand Russell

ALTERNATE INTERIOR ANGLES

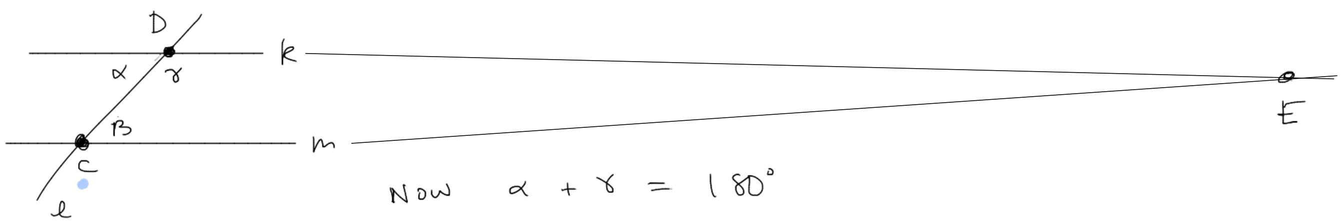


lines k, m & transversal ℓ make alt. int. angles AIA's

Prop 1: If AIA's are equal then the two lines are parallel.

proof.

Assume angle $\alpha = \beta$, ℓ is transversal on m & k.
Also assume k and m intersect, @ E



Now $\alpha + \gamma = 180°$
so $\beta + \gamma = 180°$

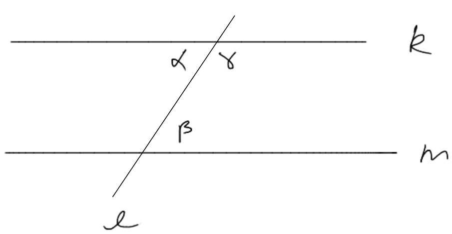
Now $\triangle DCE$ has an exterior angle α. By the Ext. Angle thm $\alpha > \beta$. (X)

so E cannot exist. $\Rightarrow$ $k \parallel m$

Note, the proof above did NOT use the parallel postulate, and is true in NEUTRAL geometry (both)

Prop (2.9) If two lines k, m are parallel then their AIA's are equal.

proof.



If $\alpha \neq \beta$ then $\alpha > \beta$
$\alpha + \gamma = 180°$
$\beta + \gamma < 180°$
$\Rightarrow$ Euclid I.V. $\Rightarrow$
same side int angle add to less than $180°$ $\Rightarrow$ lines intersect (X)

$\alpha + \gamma = 180$

As

Conclusion: $\alpha = \beta$