MA484 wk 4 – wed

Homeworks returned.
Miss #3 due to "read directions"? – resubmit OK

Hw # 2 Hints

Recall $\sqrt{2}$ is algebraic since $x^2 - 2$ is a poly with root $\sqrt{2}$.

$P(x) = x^2 - 2$

$P(\sqrt{2}) = 0$

In #8, I want to see $(\sqrt{2})^{1/2} = 2^{1/4}$ is also algebraic

For example: if $Q(x) = x^4 - 2$ then $Q(2^{1/4}) = (2^{1/4})^4 - 2 = 0$:)

Next: $(\sqrt{2})^{1/3} = (2^{1/2})^{1/3} = 2^{1/6}$

$Q_3(x) = x^6 - 2$

Idea: $Q_3(x) = P(x^3) = (x^3)^2 - 2 = x^6 - 2$

If c is algebraic then $\exists$ $P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ (i.e., $x^2 - 2$) s.t.

$P(c) = a_n c^n + a_{n-1} c^{n-1} + \dots + a_1 c + a_0 = 0$

$c^{1/2}$

$Q(x) = P(x^2) = a_n (x^2)^n + a_{n-1}(x^2)^{n-1} + \dots + a_1 x^2 + a_0$

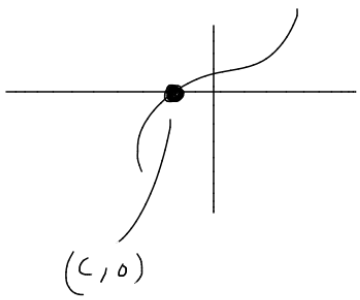
$= a_n x^{2n} + a_{n-1} x^{2n-2} + \dots + a_1 x^2 + a_0$

$Q(c^{1/2}) = a_n (c^{1/2})^{2n} + a_{n-1}(c^{1/2})^{2n-2} + \dots + a_1 (c^{1/2})^2 + a_0 = 0$

#10 Hint: c is alg $\Rightarrow \frac{1}{c}$ is alg

"c^{-1}"

$P(x) = x^3 - 3x^2 + 2$



and $R(x) = x^2 - 7x + 12$, $(x-3)(x-4)$, $R(3) = 0$

try

$Q(x) = R\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - 7\left(\frac{1}{x}\right) + 12 = \frac{1}{x^2} - \frac{7}{x} + 12$

not a poly

$Q\left(\frac{1}{3}\right) = \frac{1}{\left(\frac{1}{3}\right)^2} - \frac{7}{\left(\frac{1}{3}\right)} + 12$

$= \frac{1}{\frac{1}{9}} - \frac{7}{\left(\frac{1}{3}\right)} + 12 = 9 - 21 + 12 = 0$

know $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$ when $x = \frac{1}{3}$

mult RHS by x^2

$1 - 7x + 12x^2 = 0$ when $x = \frac{1}{3}$

gen'l case for #10

c alg $\Rightarrow$ $P(x) = \sum_{i=0}^{n} a_i x^i$

(what we did previous slide $Q(x) = x^n \cdot P(x^{-1})$

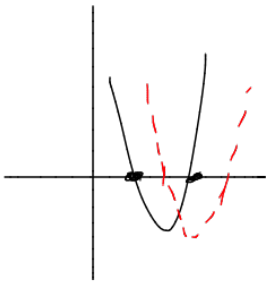
$$Q(x) = x^n \cdot P(x^{-1}) = x^n \sum_{i=0}^{n} a_i (x^{-1})^i = \sum_{i=0}^{n} a_i (x)^{\square}$$

does this pass across $\sum$?

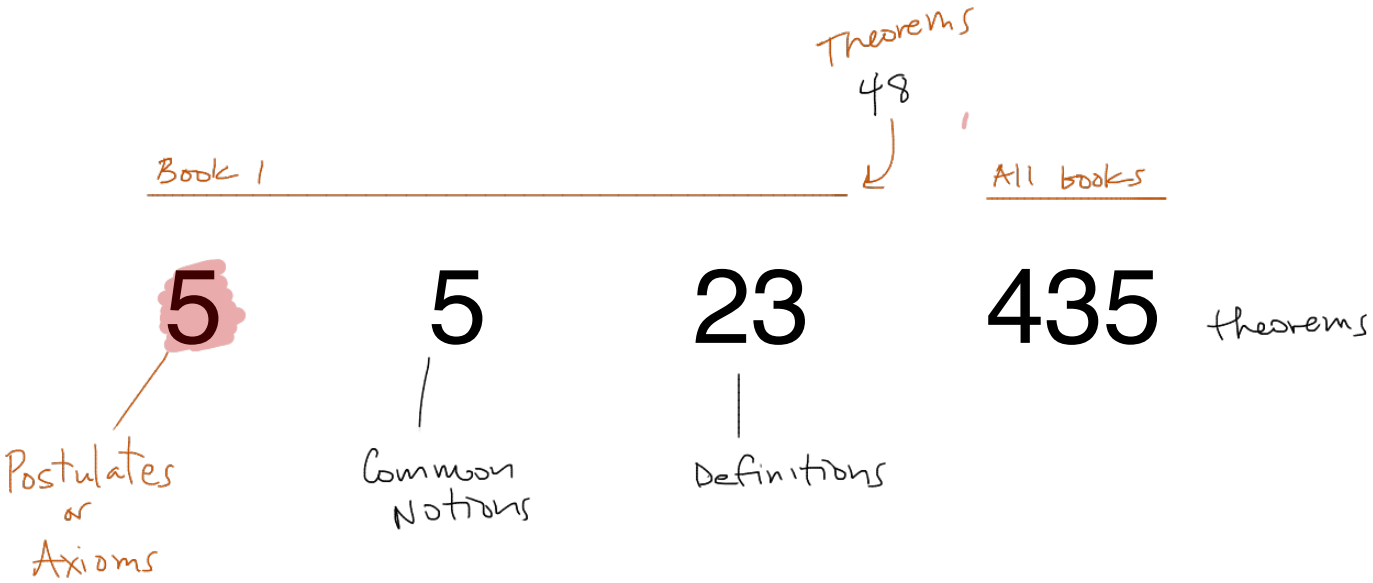
14

$x^2 - 7x + 12$, $\Rightarrow$ 3 & 4 are algebraic

show 3+1 , 4+1 are algebraic



Euclid's Elements : 13 books



Common Notions

Common notion 1.
Things which equal the same thing also equal one another.

Common notion 2.
If equals are added to equals, then the wholes are equal.

Common notion 3.
If equals are subtracted from equals, then the remainders are equal.

Common notion 4.
Things which coincide with one another equal one another.

Common notion 5.
The whole is greater than the part.