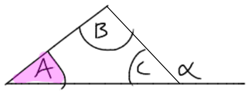
Monday

1. Homework Hints
2. Proofs from Elements Book 1
3. AIA & Hyperbolic Geometry

Last Time:

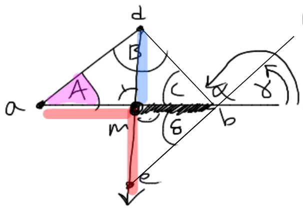
Exterior Angle thm



$\alpha > A$
or
$\alpha > B$

Neutral Geometry

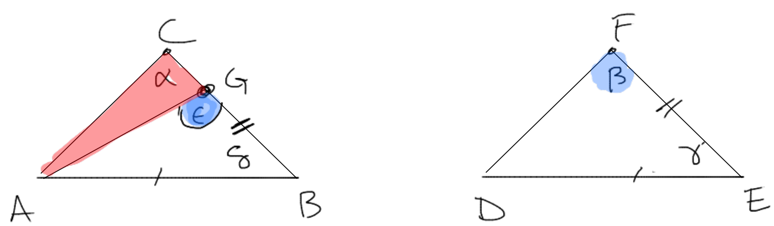
Show this: $\alpha > A$



1. m = midpt of ab
2. Form $\overrightarrow{dm}$ s.t. e on it has $|me| = |dm|$
3. Form de
4. construction + V. angles $\Rightarrow \triangle amd \cong \triangle emb$
5. $\Rightarrow \angle\delta = \angle A$
6. Finish:
   Extend eb, get γ a vertical angle of δ. ($\gamma = \delta$
   $\alpha > \gamma = \delta = A$.

AAS - theorem:

this proof could have easier, but @ cost of introducing more tools ______.



Assume

$\angle\alpha = \angle\beta$
$\angle\delta = \angle\gamma$
$AB = DE$

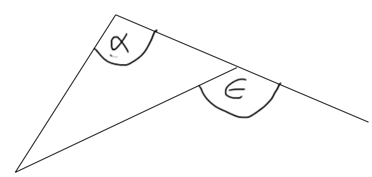
proof use ext. angle theorem

If $EF \cong BC$, done by SAS
else assume $|BC| > |EF|$, choose G on BC s.t. $BG \cong EF$

Form GA. Now $\triangle GBA \cong \triangle FED \Rightarrow \angle\epsilon = \angle\beta$

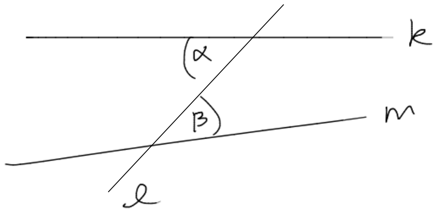
Ext. Ang thm $\Rightarrow \epsilon > \alpha$

yet $\angle\epsilon = \angle\beta = \angle\alpha$
assumption



$\Rightarrow EF \cong BC$, so by SAS, $\triangle ABC = \triangle DEF$

ALTERNATE INTERIOR ANGLES

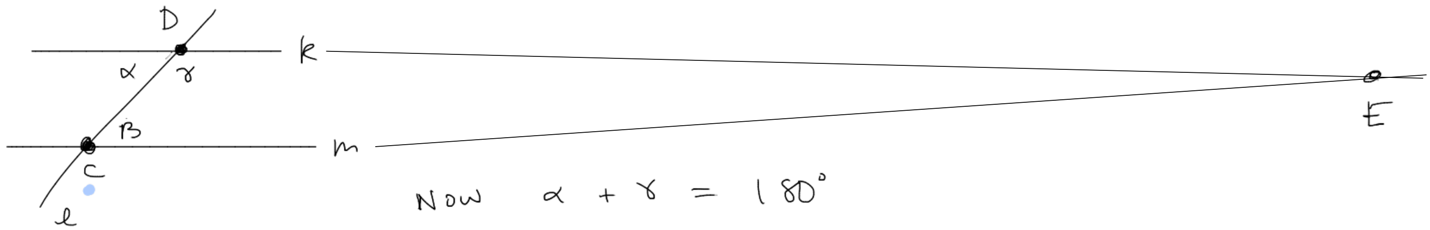


lines k, m & transversal ℓ make alt. int. angles AIA's

Prop 1: If AIA's are equal then the two lines are parallel.

proof.

Assume angle $\alpha = \beta$, ℓ is transversal on m & k.
Also assume k and m intersect, @ E



Now $\alpha + \gamma = 180^\circ$
so $\beta + \gamma = 180^\circ$

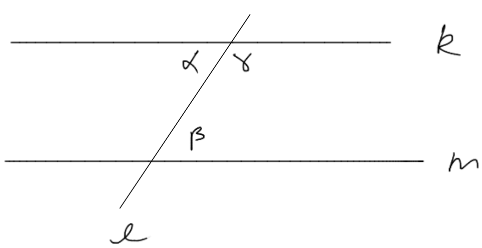
Now $\triangle DCE$ has an exterior angle α. By the Ext. Angle thm $\alpha > \beta$. ⊗

so E cannot exist. $\Rightarrow k \parallel m$

(Note, the proof above did NOT use the parallel postulate, and is true in NEUTRAL geometry (both)

Prop (2.9) If two lines k, m are parallel then their AIA's are equal.

proof.



If $\alpha \neq \beta$ then $\alpha > \beta$
$\alpha + \gamma = 180^\circ$
$\beta + \gamma < 180^\circ$
$\Rightarrow$ Euclid I.V. $\Rightarrow$
same side int angles add to less than 180° $\Rightarrow$ lines intersect ⊗

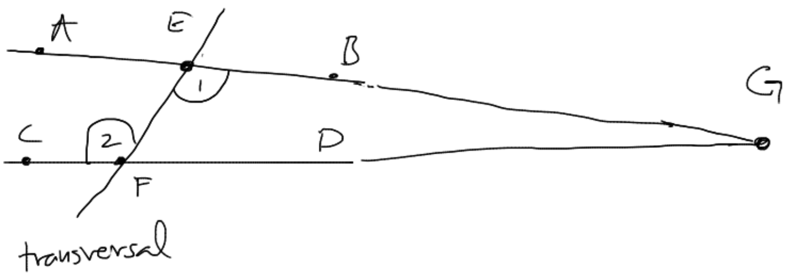
$\alpha + \gamma = 180$

Conclusion: $\alpha = \beta$

As

AIA - alternate interior angle & parallel lines

Prop. 1.27 If AIA's are = then the lines are parallel.
(true in neutral geometry (Euclidean, hyperbolic, spherical)

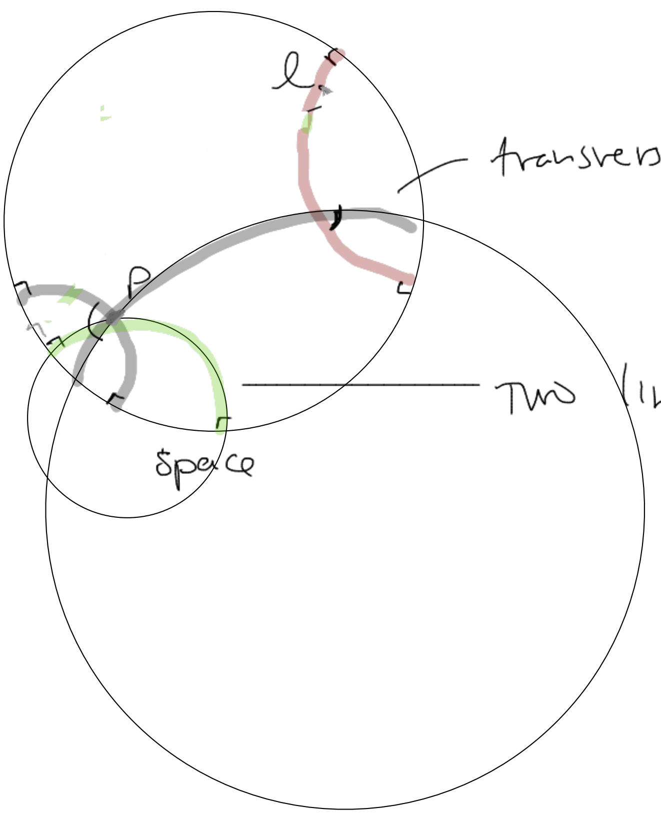


Assume $\angle 1 = \angle 2$

If AB meets CD, call that point G.
this gives $\triangle FGB$ w/ ext. angle $\angle 2$ that is = to interior angle $\angle 1$

X

Hyperbolic Geometry



Two lines thru P that are || to l

Desmos Link:

https://www.desmos.com/calculator/1zyij1hbak