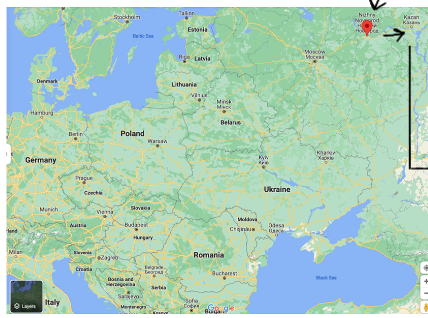
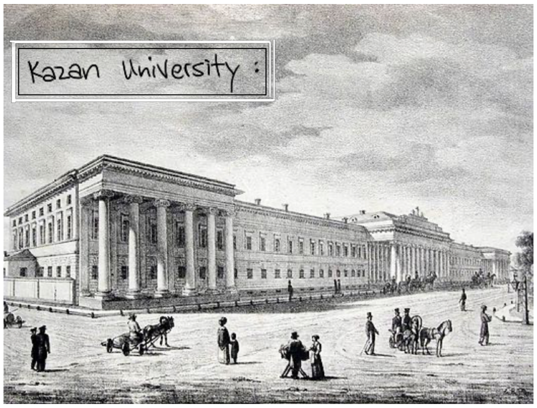
Nikolaí Lobachevsky: 1792 - 1856



Gymnasium = Prep School



—Lobachevsky enrolled at age 14—

Martin Bartels
(German)

Brought bold free-thinking ideas to Kazan

Tutored Karl F. Gauss the child prodigy before

Lobachevsky was

arrogant
rebellious
stubborn
blasphemous

1811: Age 19 →

Master's

1816:

Lobachevsky → Proffesor ... Bartels → Germany
others → Europe

math
astronomy
physics

Emperor

Alexander I

appointed → university (reviewer) M.L. Magnitsky

"spirit of dissent & irreligion"

"university should be destroyed"

"why destroy what can be corrected" → "you're in charge... fix it"

"Not a year goes by without Professor Lobachevsky willfully trying to violate our instructions… He should be closely watched."

- Magnitsky was ultimately fired
- Lobachevsky became rector

Ostrogradsky

- divergence theorem

$\iiint (\quad) \, dx\,dy\,dz = \iint \quad d\Sigma$

16. All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes — into *cutting* and *not-cutting*.

The *boundary lines* of the one and the other class of those lines will be called *parallel to the given line*.

/ Foundation Einstein's Geometry

From the point A (Fig. 1) let fall upon the line BC the perpendicular AD, to which again draw the perpendicular AE.

In the right angle EAD either will all straight lines which go out from the point A meet the line DC, as for example AF, or some of them, like the perpendicular AE, will not meet the line DC. In the uncertainty whether the perpendicular AE is the only line which does not meet DC, we will assume it may be possible that there are still other lines, for example AG, which do not cut DC, how far soever they may be prolonged. In passing over from the cutting lines, as AF, to the not-cutting lines, as AG, we must come upon a line AH, parallel to DC, a boundary line, upon one side of which all lines AG are such as do not meet the line DC, while upon the other side every straight line AF cuts the line DC.

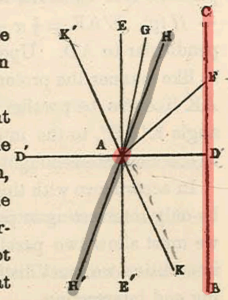


Fig. 1.

The angle HAD between the parallel HA and the perpendicular AD is called the parallel angle (angle of parallelism), which we will here designate by $\Pi(p)$ for $AD = p$.

Lobachevsky

link to Lobachevky's House Museum

link to overview slides on Lobachevky

overview

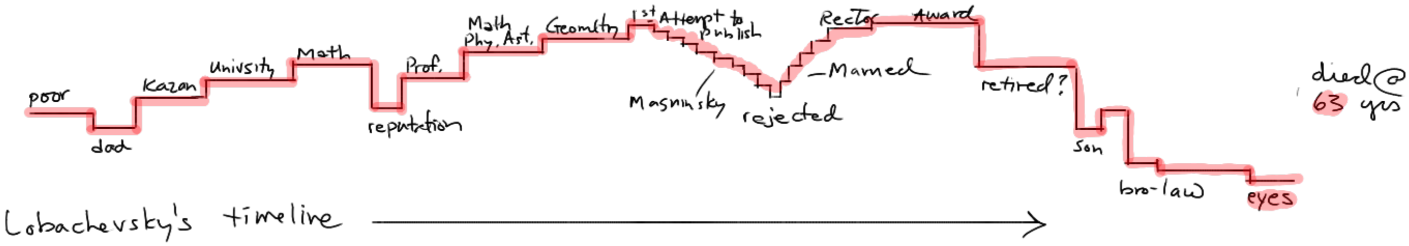
original work: theory of parallels

original work: 2

original work: 3

Martin Bartels: teacher

[redacted]



The boldness of his challenge and its successful outcome have inspired mathematicians and scientists in general to challenge other "axioms" or accepted "truths", for example the "law" of causality which, for centuries, have seemed as necessary to straight thinking as Euclid's postulate appeared until Lobachevsky discarded it. The full impact of the Lobachevskian method of challenging axioms has probably yet to be felt. It is no exaggeration to call Lobachevsky the Copernicus of Geometry, for geometry is only a part of the vaster domain which he renovated; it might even be just to designate him as a Copernicus of all thought.

On Non-Euclidean Geometry: Lobachevsky

Original Source: https://archive.org/details/in.ernet.dli.2015.165707/page/n65/mode/2up?view=theater

Prop: Parallel implies AIA are congruent

Proof: (Dunham)

Assume two lines are parallel. Therefore they don't intersect. If AIA's are congruent we're done. If not, one of the AIA's (say angle A is greater than the other (angle a). Then the sum of the supplementary angle to angle A, A', and a is less than 180.

180 = A + A' > a + A'

Postulate V implies that the two lines intersect on the side of a and A', a contradiction.

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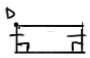
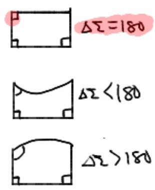
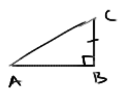
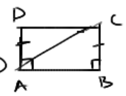
proof: (using $\Delta\Sigma = 180$ only)

lemma: $\Delta\Sigma = 180 \Rightarrow$ RAH (If you see:  then angle D is right.)

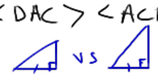
(Saccheri 1600's)



Start ① 

② 

Raise $AD \perp AB$
Find $AD = BC$

③ If $DC > AB$ then $\angle DAC > \angle ACB$
(using $AD = BC$) 

④ Compl. Angles : $\angle DAC + \angle CAB = 90$

⑤ Using ③ $90 = \angle DAC + \angle CAB > \angle ACB + \angle CAB$

⑥ In ΔABC $180 > 90 + \angle ACB + \angle CAB = \angle ABC + \angle ACB + \angle CAB$

⑦ If $DC < AB$ then $\angle DAC < \angle ACB$

similarly In ΔABC — $90 = \angle DAC + \angle CAB < \angle ACB + \angle CAB$) adding 90°

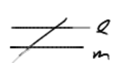
so $180 < \angle ABC + \angle ACB + \angle CAB$

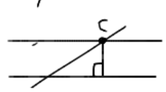
⑧ If $DC = AB$ then SSS $\Rightarrow \Delta DCA \cong \Delta BCA$ so $\angle DAC = \angle ACB$

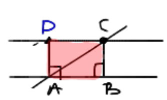
so Comp. Angles $\Rightarrow$ $90 = \angle DAC + \angle CAB$
$= \angle ACB + \angle CAB$

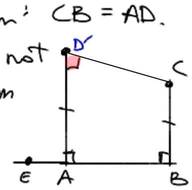
adding 90

$180 = \angle ABC + \angle ACB + \angle CAB$

Proof Assume $l \parallel m$ w/ transversal t. 

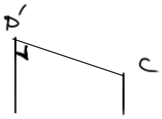
① Drop $\perp$ from C 

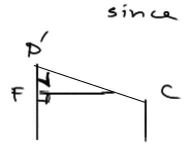
② Raise $\perp$ from A 

③ Claim: $CB = AD$.
If not lengthen 

Lemma $\Rightarrow$ $\Delta\Sigma = 180$ since then $\angle D' = 90$

④ Now $\angle EAD' = \angle CD'A$ & AIA thus $D'C \parallel AB$, giving two lines thru C $\parallel$ to AB.
(could invoke uniqueness here)

⑤ 

Raise $\perp$ from a point F along AD' (we lengthed AD) 

$\Delta\Sigma$ of $D'FC > 180$ ⊗

⑥ So $CB = AD$ & $\Delta\Sigma = 180$ gives AIA's $\cong$