The Elements ______

It's Axiomatic! System of knowledge built from a few basic truths ______.

common notions		Propositions	
5	5	23	48 435
Postulates (Axioms)		definitions	Total

Euclid's 5 Postulates:

1. Two points determine a line
2. Lines can be extended indefinitely
3. A center & radius determine a circle
4. All right angles are equal
5. (equivalently) There exists a unique parallel line to any given line through a given point.

$\exists!$ $\parallel$ line m to any line l at given pt. P.

Euclid - Book I

▼1. Preliminaries

▼a. Got to start some place

1. Point: That which has no part
2. Line: Breadthless width
3. Straight line: A line that lies evenly between its points

▼b. Modern Geometry

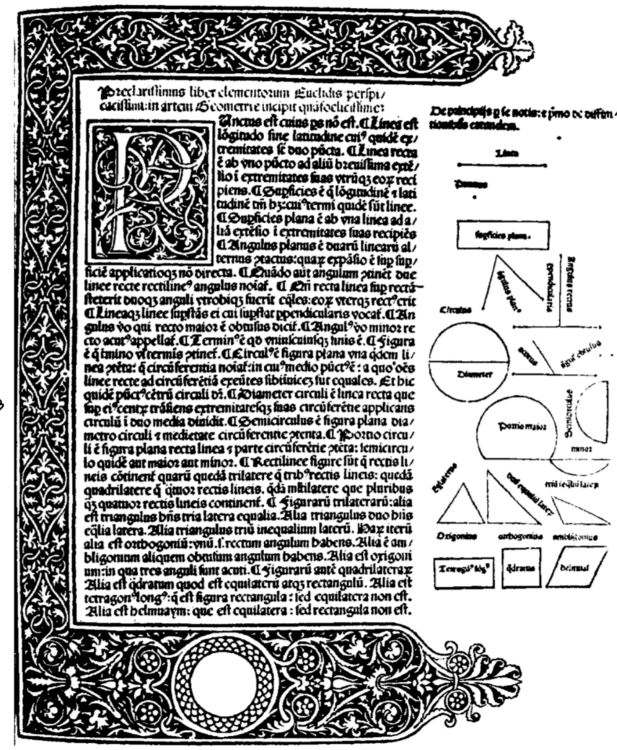
1. Point, Line are undefined terms

▼c. Other definitions

1. when a line stands on another and the adjacent angles are equal
2. No mention of angles in the Elements
3. Parallel - when two straight lines do not meet

▼d. The Postulates

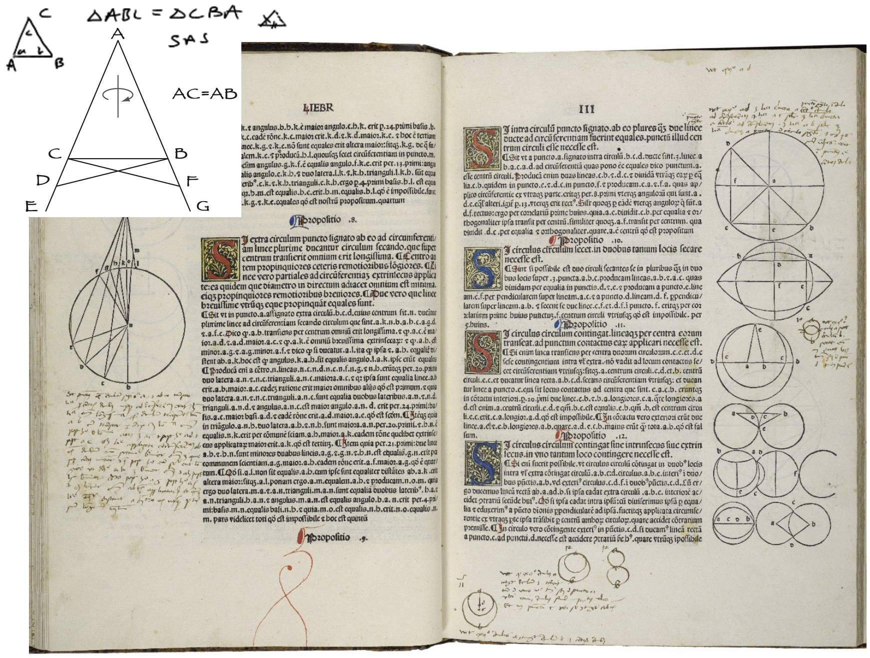
1. Two points determine a line
2. Lines can be extended arbitrarily
3. A center and distance (radius) determine a circle
4. All right angles are equal
5. If a line falling on two straight lines make the **interior angles on the same side less than two right angles**, the two straight lines, if produced indefinitely, **meet** on that side on which are the angles less than the two right angles.

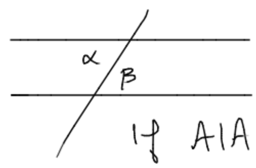


A page from the rst printed edition of Euclid's *Elements*. Published in Latin in 1482. *(Courtesy of Burndy Library.)*

Euclid's Elements - Book I

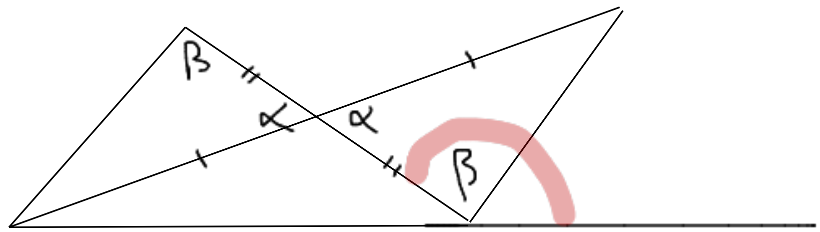
- 1. **Propositions 1 - 26**
  - a. **Triangle Congruence**
    - i. SAS
      - 1. Uses superposition - moving one object onto another to define congruence.
        - a. What is superposition?
          - i. group theory - Charles Dodgson (Lewis Carrol) would say no!
      - 2. Modern development: SAS is an axiom
    - ii. SSS & ASA follow from SAS
  - b. **Constructing perpendiculars**
  - c. **Isosceles triangles**
- 2. **Propositions 27+ : Parallels**



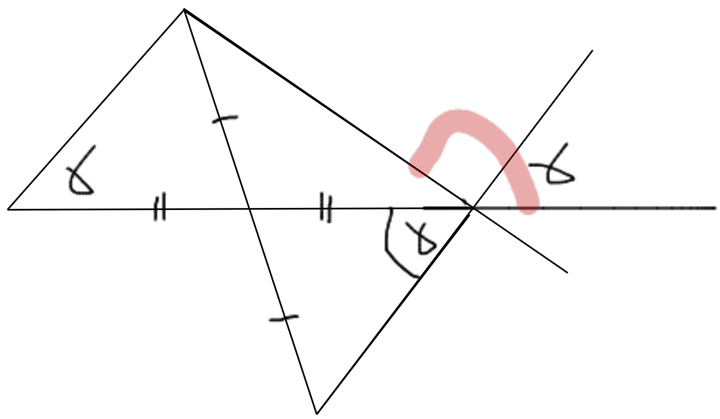


If AIA's are same, then ||

$\alpha + \beta + \gamma = 180^\circ$ uses I.V. } Euclidean Geometry.



Exterior angle Theorem

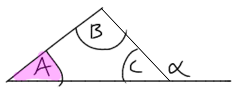


Monday

1. Homework Hints
2. Proofs from Elements Book 1
3. AIA & Hyperbolic Geometry

Last Time:

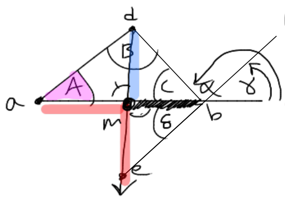
Exterior Angle thm



$\alpha > A$
or
$\alpha > B$

Neutral Geometry

show this: $\alpha > A$



1. m = midpt of ab
2. Form $\overrightarrow{dm}$ s.t. e on it has $|me| = |dm|$
3. Form de
4. construction + V. angles $\Rightarrow$ $\triangle amd \cong \triangle emb$
5. $\Rightarrow \angle\delta = \angle A$
6. Finish: Extend eb, get γ a vertical angle of δ. ($\gamma = \delta$

$\alpha > \gamma = \delta = A$.

Apply the Ext. Angle Theorem Here.

PROPOSITION I.26 (AAS) If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, . . . that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle equal to the remaining angle.

PROOF Consider Figure 2.9. By hypothesis $\angle 2 = \angle 5$, $\angle 3 = \angle 6$, and $\overline{AB} = \overline{DE}$. Euclid claimed that sides *BC* and *EF* must then be equal as well. To prove this he assumed, on the contrary, that one side was longer than the other; for instance, suppose $\overline{BC} > \overline{EF}$. It was thus possible to construct segment *BH* equal in length to *EF*. Draw segment *AH*.

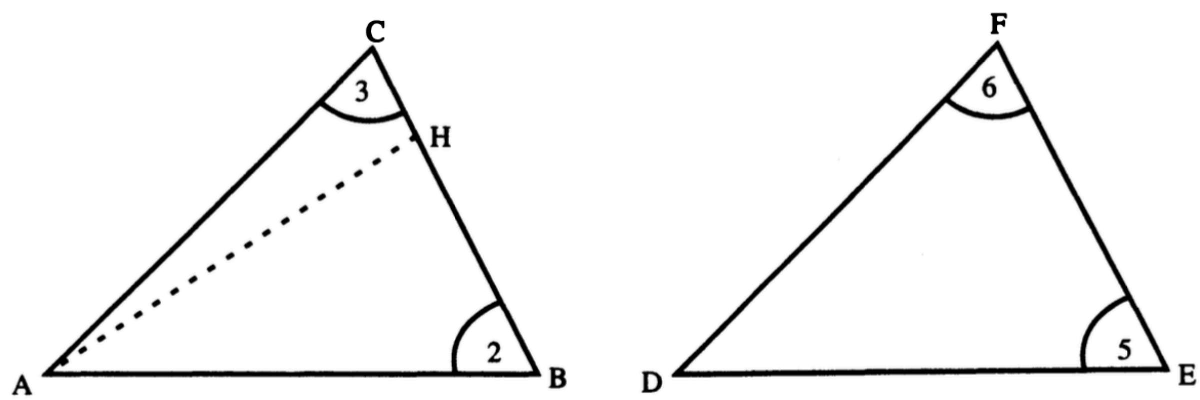


FIGURE 2.9