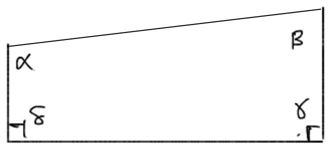
Monday - Week 5

Today: homework hints

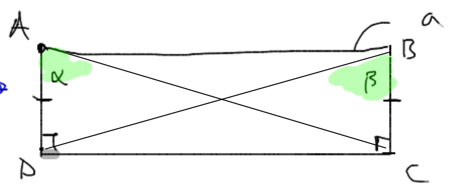
: Pentagon & Golden Ratio.

H/W: Saccheri Quadrilaterals $\alpha + \beta + \gamma + \delta < 360$



These exist in non-Euclidean (Hyperbolic) geometry

when the sides are equivalent, the summit angles are equal.



If $\overline{AD} = \overline{BC}$ then $\alpha = \beta$

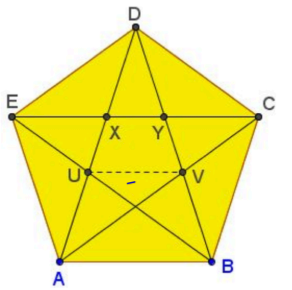
Saccheri

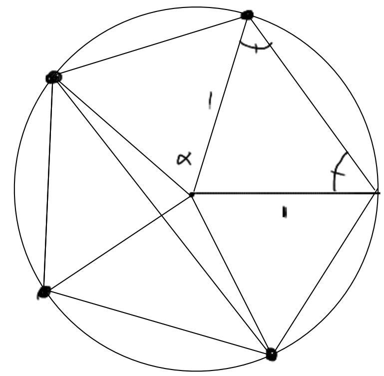
Golden Ratio in Regular Pentagon

The *golden ratio*, $\phi = \frac{1+\sqrt{5}}{2}$, makes frequent and often unexpected ***appearance in geometry***. Regular pentagon - the *pentagram* - is one of the places where the golden ratio appears in abundance.

$5\alpha = 360° = 2\pi$

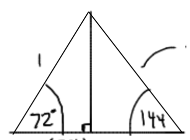
$\alpha = 72° = \frac{2\pi}{5}$





To mention a few (some of which have been proved ***elsewhere***, others are straightforward):

$$\frac{DE}{EX} = \frac{EX}{XY} = \frac{UV}{XY} = \frac{EY}{EX} = \frac{BE}{AE} = \phi.$$



$\cos(72°)$

$(1 - \cos 72°)$

$2\beta + \frac{2\pi}{5} = 2\pi$

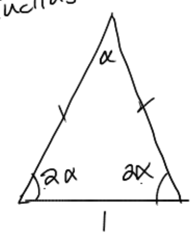
$2\beta = \frac{8\pi}{5}$

$\beta = \frac{4\pi}{5} = 144°$

$\sin\left(\frac{4\pi}{5}\right) = \frac{1 - \cos 72°}{l}$

$l = \frac{1 - \cos 2\pi/5}{\sin(4\pi/5)}$

Euclid's Isosceles



$5\alpha = 2\pi$

$\alpha = \frac{2\pi}{5}$

$\cos\left(4\frac{\pi}{5}\right) = \cos\left(2\pi - 4\frac{\pi}{5}\right) = \cos\left(6\frac{\pi}{5}\right)$

So if $x = 2\frac{\pi}{5}$:

$\cos(2x) = \cos(3x)$

Replacing the cos(2x) and cos(3x) by their general formulae:

$\cos(2x) = 2\cos^2 x - 1 \text{ and } \cos(3x) = 4\cos^3 x - 3\cos x,$

we get:

$2\cos^2 x - 1 = 4\cos^3 x - 3\cos x$

Replacing $\cos x$ by y:

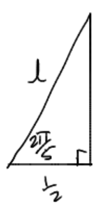
$4y^3 - 2y^2 - 3y - 1 = 0$

$(y-1)(4y^2 + 2y - 1) = 0$

We know that $y \neq 1$, so we have to solve the quadratic part:

$y = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 4 \cdot (-1)}}{2 \cdot 4}$

$y = \frac{-2 \pm \sqrt{20}}{8}$



$\cos\left(\frac{2\pi}{5}\right) = \frac{\frac{1}{2}}{l}$

$l = \frac{1}{2\cos(2\pi/5)}$

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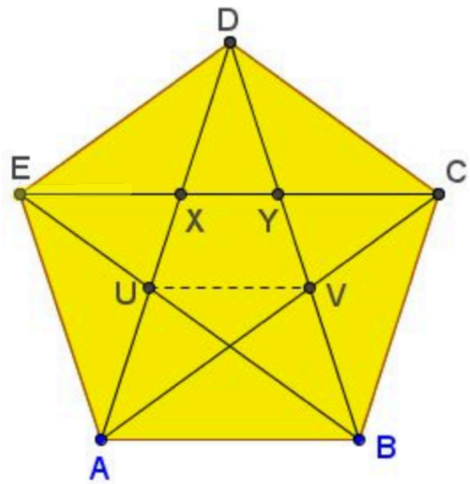

Whole = Long + Short

$$\frac{\text{Whole}}{\text{Long}} = \frac{\text{Long}}{\text{Short}}$$

To mention a few (some of which have been proved ***elsewhere***, others are straightforward):

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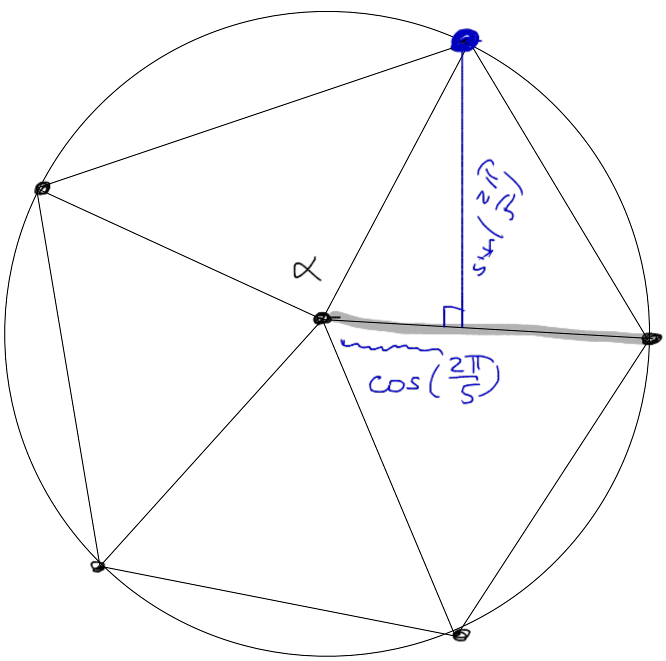
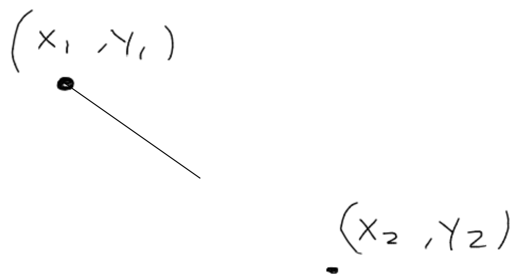
$5\alpha = 2\pi \quad \alpha = \frac{2\pi}{5}$





$(x_1(1-t)+x_2 t, y_1(1-t)+y_2 t) \quad t \in [0,1]$

$t=0 \to (x_1, y_1)$