Epilogue: The Parallel Postulate

- 1. It was thought: "Perhaps this should NOT be a postulate, but a theorem"
  - a. Postulates I-IV imply 4 triangle congruence theorems, why not this fact about parallels?
  - b. The transcendence of Euclidean geometry was as significant as when Copernicus showed us that the Earth was not the center of the universe.
    - i. "changes in conception of the cosmos"
  - c. non-Euclidean geometry is the foundation of Einstein's theory of relativity
- 2. Equivalents to I.V.
- 3. Early 1800's : three mathematicians to the rescue

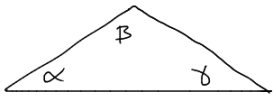
Fact: AAS, ASA, SAS, SSS are true in all geometries.

AAA $\Rightarrow$ the Δ's are similar & ratio of corresp. sides are equal

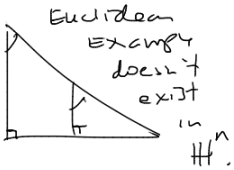
Prop. (Lobachevsky) Assuming "AAH" acute angle hypothesis, if two Δ's have identical angles they're congruent.

Hyperbolic Geom. Axiom

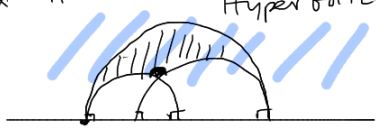
AAH: In any triangle with angles α, β, γ

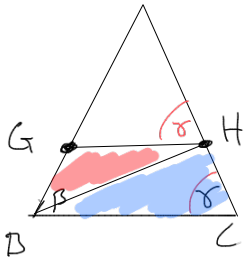


$\alpha + \beta + \gamma < 180$



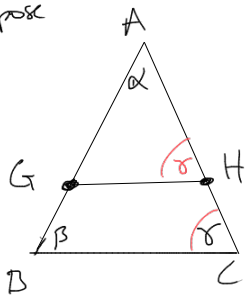
Ex $\mathbb{H}^+$ Hyperbolic triangle

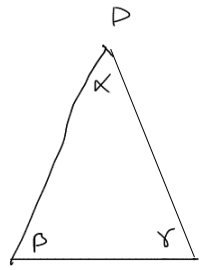




Proof

Suppose





If $|AB| = |DE|$ we're done. Else assum $|AB| > |DE|$ choose G on AB s.t. $|AG| = |DE|$, & form GH s.t. $\angle GHA = \gamma$.

Observe $\square$ GBCH contains 360°; But next decompose into Δ's. This leads to a contradiction.

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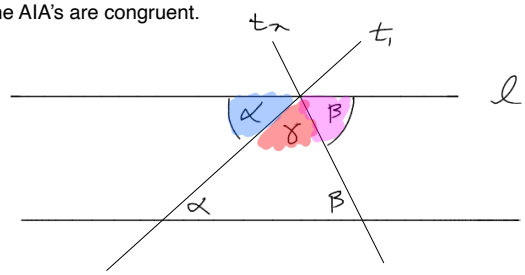
2. Equivalents to I.V.
   a. Proclus' axiom: If a line intersects one of two parallel lines, it must intersect the other
   b. Equidistance postulate: parallel lines are everywhere equidistant
   c. Playfair's postulate: Given line & point not on it, there is exactly one line through the point parallel to the line
   d. 180 degree angle sum: The sum of the interior angles of a triangle equals 180 degrees.
3. Early 1800's : three mathematicians to the rescue
   a. Gauss
      i. 
   ii. Recognized the importance of the 180 angle sum
      1. …that the angle sum of a triangle can't be less than 180 degrees … this is … the reef on which all the wrecks occur.
   iii. In 1824 he had it:
      1. "The assumption that the sum of the three angles is less than 180 leads to a curious geometry, quite different than ours, but thoroughly consistent, which I have developed to my entire satisfaction"
   iv. But 5 years later he still hadn't published his work - and had no plans to
      1. "…I fear the howl of the Boetians if I speak my opinion out loud"

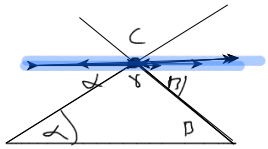
Show 180° triangle angle sum $\equiv$ I.V.

Recall that I.V. $\Rightarrow$ "Parallel lines $\Rightarrow$ AIA"

If two lines l, m are parallel, and t is a transversal, then the AIA's are congruent.



$\alpha + \beta + \gamma = 180°$



Wolfgang Farkus (father) Bolyai - 1775 - 1856

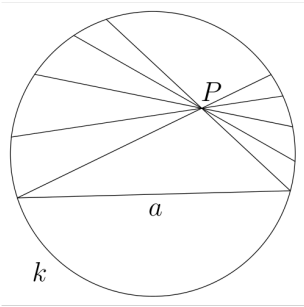
1. **Hungarian**
2. **Worked in geometry / analysis**
   - **a.** Proved a Convergence Theorem for series (something like ratio test)
   - **b.** Spent years attempting to solidify the foundations of geometry
3. **Friend of Carl Fredrick Gauss**
4. **Famous quotes**
   - **a.** *"When the time is ripe for certain things, these things appear in different places in the manner of violets coming to light in the early spring."*
     - i. This is no doubt influenced by Newton & Liebniz and likely Gauss, Johann, & Lobachevsky
   - **b.** *"You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life … I entreat you, leave the science of parallels alone."*
     - to his son , —— Farkus Bolyai

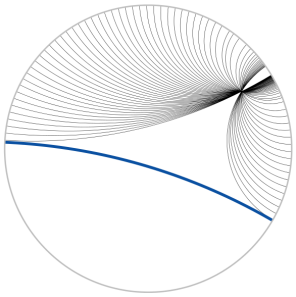
Johann Bolyai: 1802 - 1860

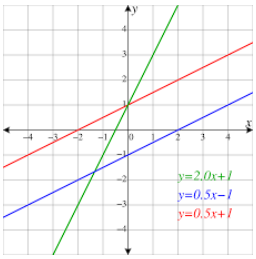
1. Despite father's wishes pushed on with parrallels
2. 
3. Realized that Euclid's parallel postulate wasn't wrong, or out of place but was IMPOSSIBLE to prove from postulates I - IV
   - a. If you replaced I.V with it's negation, you get a valid, consistent geometry
     - i. Congruence theorems still hold
     - ii. Many constructions are still possible
     - iii. No contradictions are found
4. "Out of nothing, I have created a strange new universe."
5. His father finally got behind him, included this as appendix in a paper in 1832
   - a. Sent a copy to Gauss
   - b. Gauss responded: (paraphrased) "I know this is going to sound weird, but I can't say "this is awesome" because it's exactly what I've got in my notes, and I don't want to brag, but I've known this for 35 years. I have worked this out completely already.

Other foundational geometers

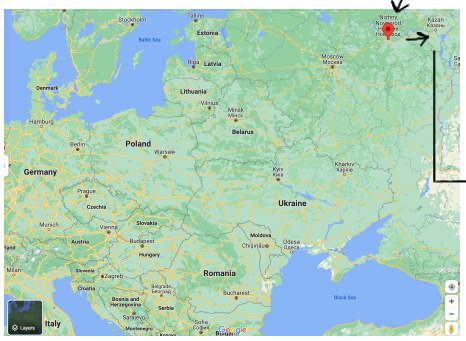
1. Nikolai Lobachevsky: 1796 - 1856
   a. Actually published similar work 3 years earlier than Bolyai in 1829
   b.
      i. 
   c. Because it was so far ahead of contemporary thought (and in Russian), it was overlooked.
   d. Age 8: Father died, Mom moved 3 sons near family in Kazan.
   e. Age 14: Scholarship to a New university w/European professors, one was Gauss' former tutor Martin Bartels
   f. Became powerful professor & rector, but clashed with those that tried to turn Kazan into a religious university
   g. Hardship: Married in his 40's, two sons died, brother in law gambled away family money, he spent money doing upgrades on University residence, started to go blind
   h. No one would support his ideas, he died @63, basically blind with no recognition
   i. Lobachevsky's teacher / Gauss' tutor is below
      i. 
2. Bernhard Riemann: 1826 - 1866
   a. 
   b. Threw out notion that lines have to be infinite
      i. Infinite lines are not implied in Euclid's postulate II
      ii. Lines are unbounded, which is different than infinite length
   c. This allowed for spherical geometry where lines act as great circles
      i. no parallel lines
3. Eugenio Beltrami: 1835 - 1900
   a.
      i. 
   b. Proved logical consistency of the different geometries
   c. Also discovered the SVD, influence tensor calculus
   d. If Euclidean geometry was consistent, then so was non-Euclidean.
      i. So Euclid's I.V was optional, not necessary.



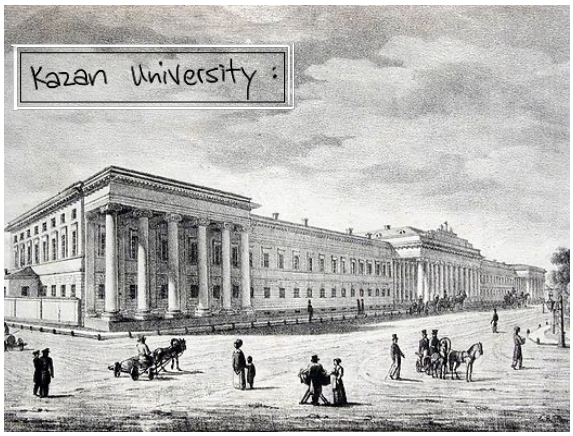




Nikolai Lobachevsky: 1792 - 1856



Gymnasium = Prep School



—Lobachevsky enrolled at age 14—

Martin Bartels (German)

Brought bold free-thinking ideas to Kazan

Tutored Karl F. Gauss the child prodigy before

Lobachevsky was

arrogant rebellious stubborn blasphemous

1811: Age 19 → Master's

1816:

Lobachevsky → Professor ... offers → Bartels → Germany

math astronomy physics

Emperor

Alexander I

appointed → university (reviewer) M.L. Magnitsky

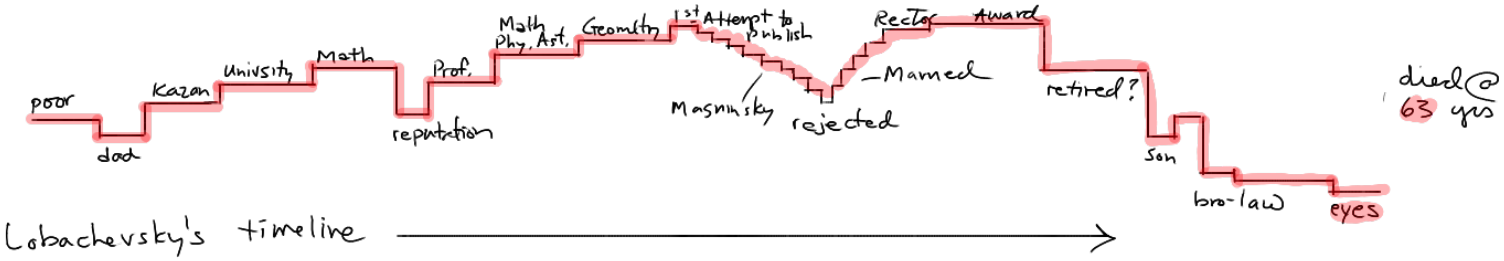
"spirit of dissent & irreligion"

"university should be destroyed"

"why destroy what can be corrected" → "you're in charge... fix it"

"Not a year goes by without Professor Lobachevsky willfully trying to violate our instructions… He should be closely watched."

- Magnitsky was ultimately fired
- Lobachevsky became rector



The boldness of his challenge and its successful outcome have inspired mathematicians and scientists in general to challenge other "axioms" or accepted "truths", for example the "law" of causality which, for centuries, have seemed as necessary to straight thinking as Euclid's postulate appeared until Lobachevsky discarded it. The full impact of the Lobachevskian method of challenging axioms has probably yet to be felt. It is no exaggeration to call Lobachevsky the Copernicus of Geometry, for geometry is only a part of the vaster domain which he renovated; it might even be just to designate him as a Copernicus of all thought.

On Non-Euclidean Geometry: Lobachevsky

Original Source: https://archive.org/details/in.ernet.dli.2015.165707/page/n65/mode/2up?view=theater

link to Lobachevky's House Museum

link to overview slides on Lobachevky

overview

original work: theory of parallels

original work: 2

original work: 3

Martin Bartels: teacher