Epilogue: The Parallel Postulate

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            1. “changes in conception of the cosmos”
    b. non-Euclidean geometry is the foundation of Einstein’s theory of relativity
2. Equivalents to I.V.
    a. Proclus’ axiom: If a line intersects one of two parallel lines, it must intersect the other
    b. Equidistance postulate: parallel lines are everywhere equidistant
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    d. 180 degree angle sum: The sum of the interior angles of a triangle equals 180 degrees.
3. Early 1800’s : three mathematicians to the rescue
    a. Gauss
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        ii. Recognized the importance of the 180 angle sum
            1. …that the angle sum of a triangle can’t be less than 180 degrees … this is … the reef on which all the wrecks occur.
        iii. In 1824 he had it:
            1. “The assumption that the sum of the three angles is less than 180 leads to a curious geometry, quite different than ours, but thoroughly consistent, which I have developed to my entire satisfaction”
        iv. But 5 years later he still hadn’t published his work - and had no plans to
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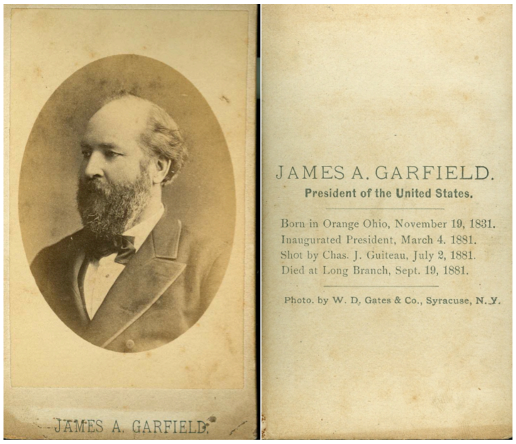
James A. Garfield: (1831 - 1881)

1. Only President to have a mathematical theorem.
2. Williams College alum
3. Taught @ Hiram College
4. Brigadier General in civil war.
5. Last of 7 Presidents to be born in log cabin.
6. First to be left handed
    a. at parties, would write simultaneously with both hands in latin & greek
7. Shot in back when he “turned down an attorney for government job”.
    a. Alexander Graham Bell had fashioned electric device to find the bullet
    b. The search for it caused infection, which killed him
8. His proof was given 5 years before he died
    a. Same year Bell invented the telephone.
    b. Published in New England Journal of Education
        i. Mistakenly (or in jest) given the latin name Pons Asinorum - Bridge of Asses



NEW-ENGLAND

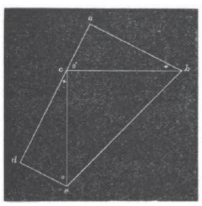
JOURNAL OF EDUCATION.

Volume III. Boston, Mass., April 1, 1876. Number 14.

Figure 3. From the title page of the *New-England Journal of Education* (Vol. 3, No.14, April 1, 1876) (image from Google Books)

PONS ASINORUM.

[In a personal interview with Gen. James A. Garfield, Member of Congress from Ohio, we were shown the following demonstration of the *pons asinorum*, which he had hit upon in some mathematical amusements and discussions with other M. C.'s. We do not remember to have seen it before, and we think it something on which the members of both houses can unite without distinction of party.]

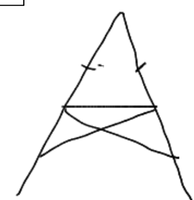


On the hypothenuse *cb* of the right-angled triangle *abc*, draw the half-square *cbe*. From *e* let fall the perpendicular *ed*, upon the side *ac* produced.

The triangles *abc* and *dce* are equal; the side $ab = dc$, and the side $ac = de$.

The area of the quadrilateral *adbe* is measured by its base *ad*, multiplied by half the sum of its parallel sides *de* and *ab*, or $ad \times \frac{\overline{ab+de}}{2}$, which is $\frac{\overline{ac+ab}^2}{2}$

But the area of the quadrilateral *adbe* consists of half of the square of *bc* plus the two equal triangles *acb* and *dce*; or $\frac{\overline{cb}^2}{2} + ab \times ac$. $\therefore \frac{\overline{cb}^2}{2} + ab \times ac = \frac{\overline{ab+ac}^2}{2}$; or $\overline{cb}^2 + 2(ab \times ac) = \overline{ab}^2 + 2(ab+ac) + \overline{ac}^2$. $\therefore \overline{cb}^2 = \overline{ab}^2 + \overline{ac}^2$. Q. E. D. [J. A. G.



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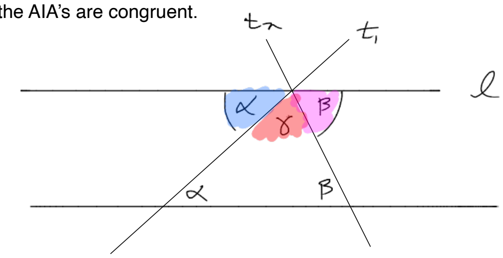
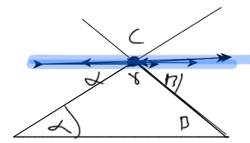
Show 180° triangle angle sum $\equiv$ I.V.

Recall that I.V $\Rightarrow$ "Parallel lines $\Rightarrow$ AIA"

If two lines l, m are parallel, and t is a transversal, then the AIA's are congruent.



$\alpha + \beta + \gamma = 180^\circ$



Wolfgang Farkus (father) Bolyai - 1775 - 1856

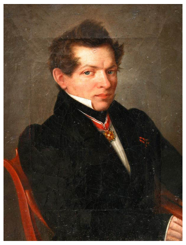
1. **Hungarian**
2. **Worked in geometry / analysis**
   a. Proved a Convergence Theorem for series (something like ratio test)
   b. Spent years attempting to solidify the foundations of geometry
3. **Friend of Carl Fredrick Gauss**
4. **Famous quotes**
   a. *"When the time is ripe for certain things, these things appear in different places in the manner of violets coming to light in the early spring."*
   b. *"You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life … I entreat you, leave the science of parallels alone."*
   - to his son , —— Farkus Bolyai

Johann Bolyai: 1802 - 1860

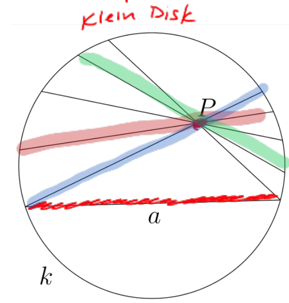
1. Despite father's wishes pushed on with parrallels
2. 
3. Realized that Euclid's parallel postulate wasn't wrong, or out of place but was IMPOSSIBLE to prove from postulates I - IV
   a. If you replaced I.V with it's negation, you get a valid, consistent geometry
4. "Out of nothing, I have created a strange new universe."
5. His father finally got behind him, included this as appendix in a paper in 1832
   a. Sent a copy to Gauss
   b. Gauss responded: (paraphrased) "I know this is going to sound weird, but I can't say "this is awesome" because it's exactly what I've got in my notes, and I don't want to brag, but I've known this for 35 years. I have worked this out completely already.

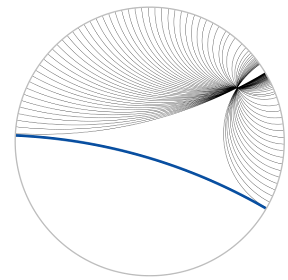
The "metaphysicians" referred to by Gauss in his letter to Taurinus were followers of Immanuel Kant, the supreme European philosopher in the late eighteenth century and much of the nineteenth century. Gauss' discovery of non-Euclidean geometry refuted Kant's position that Euclidean space is *inherent in the structure of our mind*. In his *Critique of Pure Reason* (1781), Kant declared that "the concept of [Euclidean] space is by no means of empirical origin, but is an inevitable necessity of thought." Gauss, in that letter to F. Bolyai, also wrote about " . . . the mistake Kant made in stating that space was merely the *form* of our looking at things."

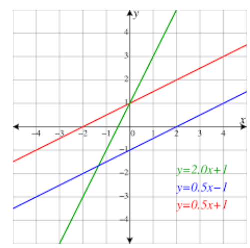
Other foundational geometers

1. Nikolai Lobachevsky: 1796 - 1856
    a. Actually published similar work 3 years earlier than Bolyai in 1829
    b.
        i. 
    c. Because it was so far ahead of contemporary thought (and in Russian), it was overlooked.
    d. Age 8: Father died, Mom moved 3 sons near family in Kazan.
    e. Age 14: Scholarship to a New university w/European professors, one was Gauss' former tutor Martin Bartels
    f. Became powerful professor & rector, but clashed with those that tried to turn Kazan into a religious university
    g. Hardship: Married in his 40's, two sons died, brother in law gambled away family money, he spent money doing upgrades on University residence, started to go blind
    h. No one would support his ideas, he died @63, basically blind with no recognition
    i. Lobachevsky's teacher / Gauss' tutor is below
        i. 
2. Bernhard Riemann: 1826 - 1866
    a. 
    b. Threw out notion that lines have to be infinite
        i. Infinite lines are not implied in Euclid's postulate II
        ii. Lines are unbounded, which is different than infinite length
    c. This allowed for spherical geometry where lines act as great circles
        i. no parallel lines
3. Eugenio Beltrami: 1835 - 1900
    a.
    b. Proved logical consistency of the different geometries
    c. Also discovered the SVD, influence tensor calculus
    d. If Euclidean geometry was consistent, then so was non-Euclidean.
        i. So Euclid's I.V was optional, not necessary.

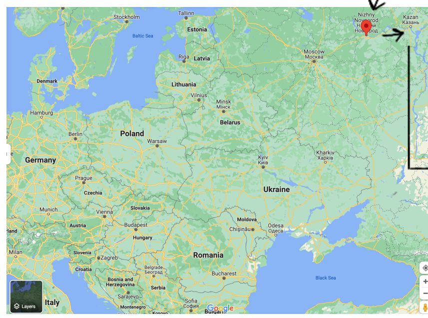
open Klein Disk



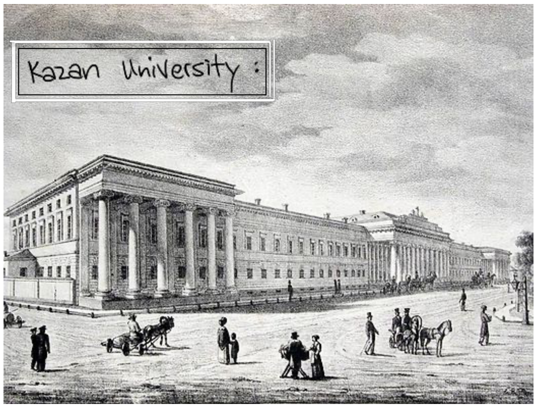




Nikolaí Lobachevsky: 1792 - 1856



Gymnasium = Prep School



— Lobachevsky enrolled at age 14 —

Martin Bartels
(German)

Brought bold free-thinking ideas to Kazan

Tutored Karl F. Gauss the child prodigy before

Lobachevsky was

arrogant
rebellious
stubborn
blasphemous

1811 : Age 19 →

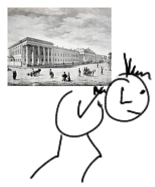
Master's

1816 :

Lobachevsky → Proffesor ... Bartels

others → Europe

Bartels → Germany



math
astronomy
physics

Emperor

Alexander I

appointed → university (reviewer) M.L. Magnitsky

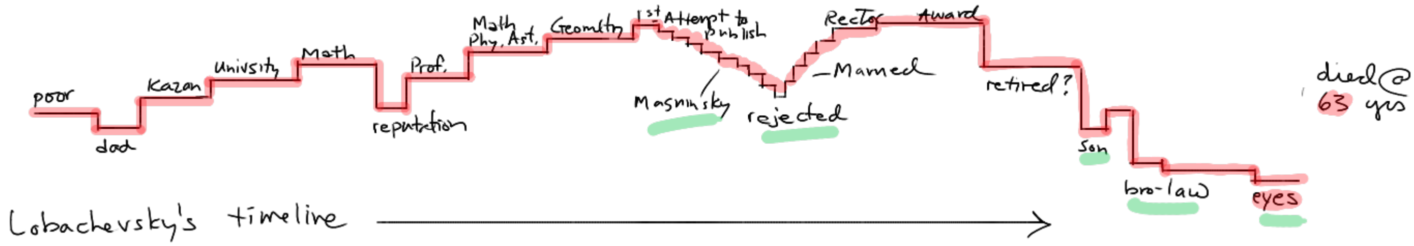
"spirit of dissent & irreligion"

"university should be destroyed"

"why destroy what can be corrected" → "you're in charge... fix it"

"Not a year goes by without Professor Lobachevsky willfully trying to violate our instructions… He should be closely watched."

- Magnitsky was ultimately fired
- Lobachevsky became rector



The boldness of his challenge and its successful outcome have inspired mathematicians and scientists in general to challenge other "axioms" or accepted "truths", for example the "law" of causality which, for centuries, have seemed as necessary to straight thinking as Euclid's postulate appeared until Lobachevsky discarded it. The full impact of the Lobachevskian method of challenging axioms has probably yet to be felt. It is no exaggeration to call Lobachevsky the Copernicus of Geometry, for geometry is only a part of the vaster domain which he renovated; it might even be just to designate him as a Copernicus of all thought.

On Non-Euclidean Geometry: Lobachevsky

Original Source: https://archive.org/details/in.ernet.dli.2015.165707/page/n65/mode/2up?view=theater

[link to Lobachevky's House Museum](#)

[link to overview slides on Lobachevky](#)

[overview](#)

[original work: 2](#)

[original work: 3](#)

[Martin Bartels: teacher](#)

Arithmeticae Ⓣ and Laplace's *Mécanique Céleste* Ⓣ. In 1814 it was mainly due to Bartels that Lobachevsky was appointed as an assistant professor. We should note that Lobachevsky took Bartels' course on the History of Mathematics which, following Montucla, considered in detail Euclid's *Elements* and his theory of parallel lines. It was this course which made Lobachevsky think about non-euclidean geometry.

Married , 7 kids , (12?)

hosted parties in the house

he invested money into the house

eyesight

son (mini-me) died unexpectedly

brother in-law gambled into debt

blind at death

Ostrogradsky

· divergence theorem

$$\iiint (\quad) dx\,dy\,dz = \iint \quad d\Sigma$$

16. All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes — into *cutting* and *not-cutting*.

The *boundary lines* of the one and the other class of those lines will be called *parallel to the given line*.

From the point A (Fig. 1) let fall upon the line BC the perpendicular AD, to which again draw the perpendicular AE.

In the right angle EAD either will all straight lines which go out from the point A meet the line DC, as for example AF, or some of them, like the perpendicular AE, will not meet the line DC. In the uncertainty whether the perpendicular AE is the only line which does not meet DC, we will assume it may be possible that there are still other lines, for example AG, which do not cut DC, how far soever they may be prolonged. In passing over from the cutting lines, as AF, to the not-cutting lines, as AG, we must come upon a line AH, parallel to DC, a boundary line, upon one side of which all lines AG are such as do not meet the line DC, while upon the other side every straight line AF cuts the line DC.

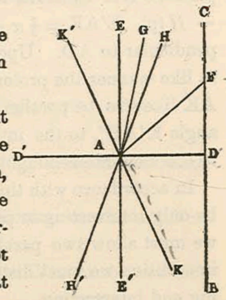


FIG. 1.

The angle HAD between the parallel HA and the perpendicular AD is called the parallel angle (angle of parallelism), which we will here designate by Π (p) for AD = p.

Foundation

Einstein's Geometry

Lobachevsky