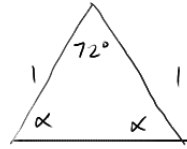


radius = 1



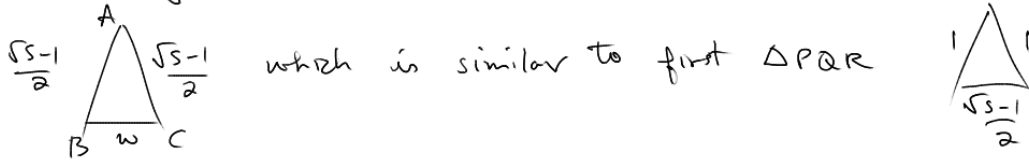
$$72 + 2\alpha = 180$$

$$\alpha = 54$$

$$72^\circ - 54^\circ = 18^\circ$$

$$\text{part (a)} \Rightarrow x = \frac{\sqrt{5}-1}{2}$$

- ① Start by in-laying unit-length isosceles  $36^\circ$   $\Delta$  @ center, 10 times to get decahedron.
- ② It's base length is  $\frac{\sqrt{5}-1}{2}$  by (a)
- ③ Connect every other vertex for pentagon, making  $18^\circ$  angle. Double it to get  $\Delta ABC$ .



$$\textcircled{4} \text{ Similar } \Delta\text{'s} \Rightarrow \frac{\frac{\sqrt{5}-1}{2}}{w} = \frac{1}{\frac{\sqrt{5}-1}{2}} \Rightarrow w = \left(\frac{\sqrt{5}-1}{2}\right)^2 = \frac{5-2\sqrt{5}+1}{4} = \frac{3-\sqrt{5}}{2}$$

$$\textcircled{5} \text{ Finally, } w \text{ is twice } QQ', \text{ so } QQ' = \frac{3-\sqrt{5}}{4}$$

$$\textcircled{6} \text{ So Pythag.} \Rightarrow (Q'R)^2 = \left(\frac{\sqrt{5}-1}{2}\right)^2 - \left(\frac{3-\sqrt{5}}{4}\right)^2 = \frac{5-2\sqrt{5}+1}{4} - \frac{(9-6\sqrt{5}+5)}{16}$$

$$= \frac{24-8\sqrt{5}}{16} - \frac{14+6\sqrt{5}}{16} = \frac{10-2\sqrt{5}}{16}$$

$$Q'R = \sqrt{\frac{10-2\sqrt{5}}{16}} = \sqrt{\frac{5-\sqrt{5}}{8}} = \frac{\sqrt{5-\sqrt{5}}}{\sqrt{8}} = \frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}} = \frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}$$

⑦ and  $Q'R$  is exactly  $\frac{1}{2}$  of side of inscribed pentagon

$$RR' = \sqrt{\frac{5-\sqrt{5}}{2}}$$