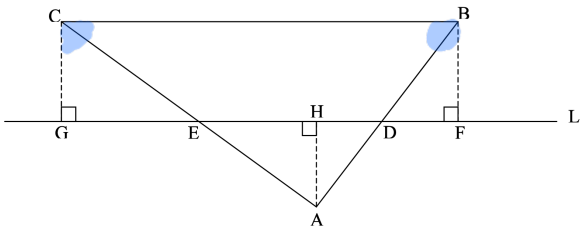
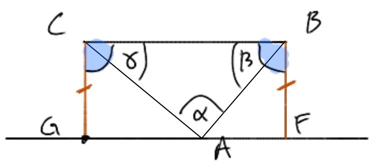


(d) Finally, prove that the sum of the measures of the three angles in ΔABC is just $\angle FBC + \angle GCB$.

33. Now prove that under HAA, the angle sum of a triangle must be less than 180°.

Let ΔABC be given & assume HAA. From #32(a) - (d) $\Rightarrow$ $\square FBCG$ is Saccheri

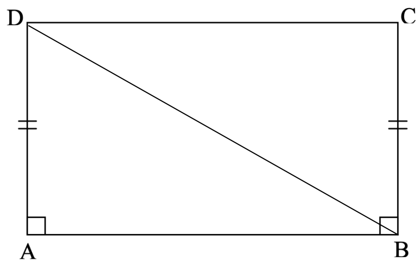


$\Rightarrow$ Saccheri have acute summit angles

$\Rightarrow$ $\alpha + \beta + \gamma = \angle FBC + \angle GCB < 90 + 90$
< 180

31. Under HAA, prove that if ABCD is a Saccheri Quadrilateral as shown, then $\overline{AB} \neq \overline{CD}$.

How to use HAA?
- Assume it holds, won't appear until the end via proof by contradiction.



proof:

Assume HAA $\Rightarrow$ $\angle ADC < 90$, $\angle BCD < 90$

ABWOC, assume, by way of contradiction, that $\overline{DC} = \overline{AB}$.

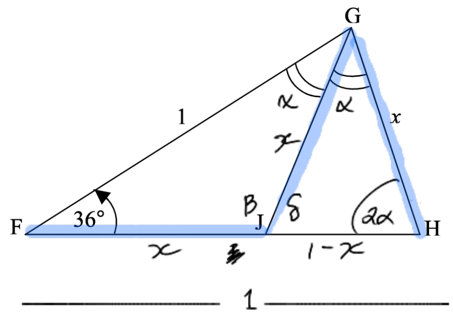
Construct: DB diagonal, get two congruent Δ's via $\underbrace{\text{usual congruence theorems}}_{\text{still hold in HAA}}$

$\Rightarrow$ corresponding angles are $\cong$ $\otimes$

(a) For future reference, consider isosceles ΔFGH where $\overline{FG} = \overline{FH} = 1$ and $\overline{GH} = x$. Suppose also that $\angle GFH = 36°$. Construct GJ bisecting $\angle FGH$. Use similar triangles to prove that $x = \frac{\sqrt{5}-1}{2}$.

Key: Pons Asinorum: Isosceles $\Delta \Rightarrow$

- two angles $\cong \Leftrightarrow$ opposite sides $\cong$
- and converse



- Euclidean $\Rightarrow \Delta\Sigma = 180$

$$\left.\begin{aligned} 36 + \alpha + \beta &= 180 \\ \delta + \beta &= 180 \\ \delta + 3\alpha &= 180 \end{aligned}\right\}$$

$\delta = 2\alpha \Rightarrow \Delta GJH$ isosceles

$\Rightarrow \overline{GJ} = x$

& $\alpha = 36°$

$\Rightarrow \Delta FJG$ isosceles

$\Rightarrow \overline{FJ} = x$

So...

- $\overline{JH} = 1 - x$

Similar Δ's:

$\Delta FHG \sim \Delta GJH$

$\frac{1}{x} = \frac{\text{long}}{\text{short}} = \frac{\text{long}}{\text{short}} = \frac{x}{1-x}$

cross-mult & solve $\quad x^2 = 1 - x \quad \ldots \quad x = \frac{\sqrt{5}-1}{2}$

44. Explain how you could find 100 consecutive numbers, none of which is prime. How about a billion consecutive non-primes? (HINT: Factorials!)

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$5! + 5 = 5(4! + 1)$

I claim: $120 + 1$, $120 + 2$, $120 + 3$, 12

121, 122, 123, 124, 125 — none are prime

$5 \cdot 4 \cdot 3 \cdot 2 + 4$
$4(5 \cdot 3 \cdot 2 + 1)$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 1$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 2$
$=$
$2(5 \cdot 4 \cdot 3 + 1)$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 3$
$=$
$3(5 \cdot 4 \cdot 2 + 1)$

4 consecutive #'s not prime

So to get 5 consecutive non-primes

$6! + 2, \; 6! + 3, \; 6! + 4, \; 6! + 5, \; 6! + 6$

1 2 3 4 5