Wed - Week 6

#46

Goldbach conjecture:

any even number ≥ 4 is expressed as sum of two primes

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- Dunham's 46^{th} conjecture

  Any integer greater than 5 is expressed as sum of 3 primes.

Assume Goldbach:

Let n be any integer greater than 5.

Case 1: n is even (Goldbach applies) (almost works: $n = P_1 + P_2$ $\square$

$n-2$ is also even ... 2 is prime

Case 2: n is odd

subtract some # where Goldbach applies

Homework Hint:

Consider the number 10

$$10 = 2 \cdot 5$$

Add 2 to 10, I know that $2 + 10$ is not prime

$$2 + 10 = 2 + 2 \cdot 5 = \underbrace{2(1+5)}_{\text{composite}}$$

$5 + 10$ is composite:

$$5 + 5 \cdot 2 = 5(1+2)$$

For factorials

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$5! + 2 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 + 2 = \underbrace{2(5 \cdot 4 \cdot 3 + 1)}_{\text{composite}}$$

Hints!

Any (integer) number has the form

$6k$, $6k+1$, $6k+2$, $6k+3$, $6k+4$, $6k+5$

 $6k-3$ $\quad$ $6k-2$ $\quad$ $6k-1$

$17 = 6(2)+5$

$ = 6(3)-1$

Finish Ch. 3 —— Monday: Start Ch. 4 (Wed.) HW due)

Mersenne Primes / $\underbrace{\text{Perfect Numbers}}$ — Sum of its proper divisors

$6 = 1+2+3$

$28 = 1+2+4+7+14$

primes of this form

$\boxed{2^p - 1 \quad (p = \text{prime})}$

Ex
$p=2 \Rightarrow 2^2-1 = 3$
$p=3 \Rightarrow 2^3-1 = 7$
$p=4 \Rightarrow 2^4-1 = 15$ (not prime)
$p=5 \Rightarrow 2^5-1 = 31$

Euclid gave an algorithm to generate perfect #'s.

Form: $1+2+2^2+2^3+\dots+2^n$

If this is prime, multiply by 2^n

$2^n(1+2+2^2+2^3+\dots+2^n)$

then this is perfect

Why are these so perfect?

let $m = 2^n(\underbrace{1+2+2^2+2^3+\dots+2^n})$.

If this is prime then it's easy to write down the factors of n:

1. $1+2+2^2+\dots+2^n$
2. 1
3. 2
4. 2^n
5. $2(1+2 \dots 2^n)$
6. $2^k(1+2+\dots+2^n)$ w/ $1<k<n$

Recall Euclid's Lemma:
If a prime p divides ab then it must divide either a or b.

expanding the RHS

$m = (1+\dots+2^n) + 2(1+\dots+2^n) + 2^2(1+\dots+2^n) + \dots + 2^{n-1}(1+\dots+2^n)$

What's the connection b/w

$$2^p - 1 \quad \stackrel{?}{=} \quad 1 + 2 + 2^2 + \dots + 2^n.$$

Mersenne
Prime

Turns out:

$$\underline{2^{ab} - 1} = (2^a - 1)(1 + 2^{2} + 2^{2a} + 2^{3a} + \dots + 2^{(b-1)a})$$

For ex:

$$\left.\begin{array}{l} a = 2 \\ b = 2 \end{array}\right\} \; 2^4 - 1 = 15 = (2^2 - 1)(1 + 2^{(2-1)\cdot 2})$$

$$3 \cdot (1 + 2^2) = 3 \cdot 5$$

Every perfect # ever found has Euclid's Form.

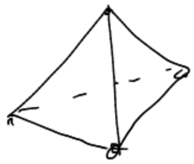
No odd perfect #'s have been found ——.

As of 2006: No odd perfect # less than 10^{1500}

Platonic Solids:

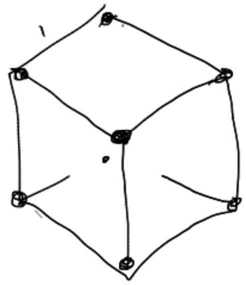
$$V - E + F$$

# of vertices   # edges



$4 - 6 + 4$

$= 2$



$8 - 12 + 6 = 2$