

# 1 General Questions

1. Who invented mathematics?
2. Who is often considered the first mathematician?
3. Compare Egyptian, Greek, Babylonian & Islamic mathematics of the Classical Period.
4. Show that  $\sqrt{5}$  is algebraic.

5. Is  $\pi$  algebraic? What does this question have to do with "squaring the circle"?

6. Show that  $\pi^2$  is not algebraic.

7. Are most of the results in the Elements due to Euclid?

8. Discuss the Parallel Postulate and the dramatic story surrounding it.

9. Discuss the story of the discovery of non-Euclidean geometry as it relates to Carl Fredrick Gauss and the Bolyai family.

10. What figure did Euclid use in his proof of the fact, that would today be known as  $(a + b)^2 = a^2 + 2ab + b^2$ .

11. Be able to recognize the regular Platonic solids, describe why there are only five, and what the Euler Characteristic indicates about them.

12. Discuss  $\pi$ . How many digits of accuracy was Archimedes' approximation of  $\pi$ ? How did he find this approximation? How many digits of  $\pi$  were accurately computer before the use of computers - 35, 100, 800, 2000?

## 2 People

### 1. Thales of Miletus

- (a) When and where did he live?
- (b) What is known about his personality?
- (c) Why didn't he marry?
- (d) What are some mathematical insights due to him?

### 2. Pythagoras

- (a) When and where did he live?
- (b) What are several things the Pythagoreans are known for?
- (c) What theorem reportedly caused a Pythagorean to be murdered at sea?
- (d) Why were the Pythagoreans so troubled by this result?

### 3. **Euclid**

(a) When and where did he live?

(b) What is he known for?

(c) Discuss the Elements, its impact and the topics within its various books

### 4. **Hippocrates**

(a) When and where did he live?

(b) What mathematical result is due to him?

(c) What is the basic idea behind this result?

## 5. Eudoxus

(a) What two major contributions to mathematics did he make?

(b) At what famous place did he study?

## 6. Archimedes

(a) When and where did he live?

(b) Recount two or more stories surrounding Archimedes.

(c) What pieces of mathematics and/or engineering is he famous for?

## 7. Ramanujan

(a) When and where did he live?

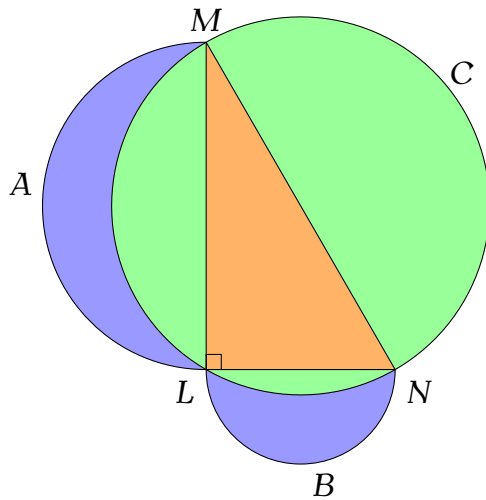
(b) Ramanujan is introduced *out of order* because of his results on  $\pi$ . He discovered the following interesting formula for computing  $\pi$

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n + 5}{2^{12n+4}}.$$

Work out the first three terms of this series to compute an approximation of  $\pi$ . How many digits of accuracy are achieved by just three terms?

(c) Describe some interesting facets of Ramanujan's life.

8. Each edge of the triangle below is a diameter of the adjacent semi-circle.



(a) Use the Pythagorean Theorem to show the areas of the semicircles are as follows:

$$\triangle A + \triangle B = \triangle C.$$

(b) Use Hippocrates' ideas to show the combined area of the lunes in the figure below is the same as the area as the triangle:

$$\text{Lune}(MAL) + \text{Lune}(LBN) = \text{Triangle}(MLN)$$



### 3 Great Theorems

(a) What does it mean for a plane figure to be quadrable - or equivalently to square an object.

(b) Is it possible to square: any lune, every lune, a circle?

(c) Show explicitly how to square a rectangle.

(d) Give a proof of the Pythagorean theorem.

(e) Sketch a proof that there are an infinite number of primes.

(f) Describe the basic idea behind Archimedes' approximation of the circumference of a circle.