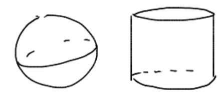
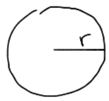
Monday - Week 7

1. Archimedes: 225 BC
    a. Works
        i. On the Measurement of a Circle
            1. Showed how to calculate the area of a circle
                a. by relating it to a triangle
            2. Estimated pi (well) by exhaustion
                a. It had been estimated before his time: I Kings 7:23
                    i. "Then He made the molten sea, then cubits from brim to brim, while a line of 30 cubits measured it round."
        ii. On the Sphere & Cylinder
            1. Computed areas of spheres / cones / cylinders
            2. Related the sphere to the cylinder in interesting way
            3. Related the volume constant, the area constant, the length constants … all to what would be known as pi
            4. Prop. 13 The surface of any right circular cylinder excluding bases is equal to a circle whose radius is a a mean proportional between the side of the cylinder and the diameter of the base.
                a. Explaination: "mean proportional" b/w side of cylinder and diameter. let h = side of cylinder, 2r = diameter, the mean proportional is x where h/x = x/(2r).
                    i. Thus x^2 = 2hr and the radius is x. Thus the area of the open cylinder is the same as the area of the circle with radius x. We know this to be pi * x^2 thus 2*pi*r*h

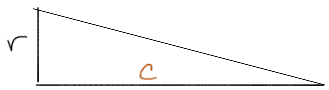


Arch's Proof of how to calculate area of a circle

- Given



circle, radius r



& triangle, height = r, circumference = C base

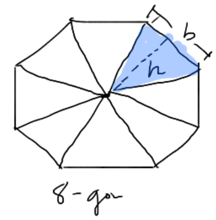
Proof:

A = area of circle,
T = area of triangle

CLAIM: A = T ?

Lemma: Area of regular n-gon

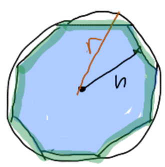
$A = n \cdot \frac{1}{2} b \cdot h = \frac{n}{2} \cdot b \cdot h$?



Perimeter of a regular n-gon

$P = n \cdot b$

case 1 $\underbrace{A - T}_{\epsilon} > 0$. Archy inscribes a regular n-gon polygon inside circle. ($\Rightarrow h < r$, Perimeter $< C$; $= nb$)



By increasing the # of sides we can get the area of the polygon P, as close as we want to the area of the circle

So, $A - P < \epsilon = A - T \Rightarrow$ $\boxed{T < P}$

$\frac{1}{2} C r = \frac{1}{2} b h < \frac{n}{2} bh$

$Cr < nbh$
but $C > nb$
and $r > h$
⊗

case 2 $A - T < 0 \Rightarrow \overbrace{T - A}^{\text{small}} > 0$

Circumscribe

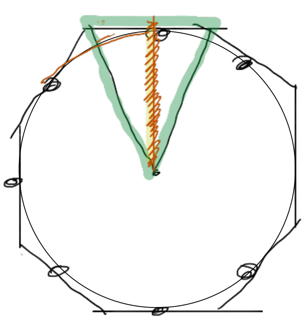
$\Rightarrow$ $h = r$
$\Rightarrow$ Perimeter $= nb > C$

So increase # sides s.t.

$\underbrace{P - A}_{\text{smaller}} < T - A \Rightarrow P < T$ = area of triangle = $\frac{1}{2} r C$

$\frac{n}{2} bh$ = area of poly

$\Rightarrow$ contradiction!



Pi

1. Egypt - Rhind Papyrus - (4/3)^4 = 3.160..
2. Bible: I Kings 7:23
   a. "Then He made the molten sea, ten cubits from brim to brim, while a line of 30 cubits measured it around."
3. In the 2nd century CE, Ptolemy used the value $^{377}/_{120}$, the first known approximation accurate to three decimal places. It is equal to 3 + 8/60 + 30/60^2
   a. table of chords
      i. the chord of 1 degree is 1.0472 p where the diameter is 120 p
      ii. multiplying the above by 360 gives the circumference: 376.992p or pi = 3.1416
4. Chinese: 150 AD
   a. The Chinese mathematician Liu Hui in 263 CE computed π to between 3.141024 and 3.142708 by inscribing a 96-gon and 192-gon;
5. Bhaskara (1110 CE) … pi = 3.1416
6. Simon Stevin (1500 CE) - decimal system, helped matters
7. Francois Viete: (1550 CE)
   a. used polygons with 393,216 sides … 9 decimal places.
   b. Ludolph van Ceulen (1600's)
      i. 35 correct decimal places
      ii. after years of effort
      iii. polygon with 2^62 sides. (4 million trillion sides)
   c. Leibniz's series: 1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 + … approximates pi/4
      i. from geometry to arithmetic
      ii. approaches slowly
      iii. after 150 terms only get 3.1349
      iv. little practical use
   d. Shart (1650) 71 places, Machin (1680) 100 places
   e. Lambert (1750) pi is irrational, so no decimal is gonna get it.
8. Ramanujan (1887-1920)
   a. poor, self taught, failed out of school,
   b. was urged to write of his discoveries to England, one sent to G.H. Hardy (Cambridge) (1913)
   c. strange formulas, poor English … it haunted Hardy all day
   d. travel to England was hard due to religion, diet, but he arrived in Cambridge 1914.
   e. Highly accurate approximations to pi
   f. 1919, back to India in poor health.
   g. Story of Ramanujan on death bed
      i. The number 1729 is known as the Hardy–Ramanujan number after a famous visit by Hardy to see Ramanujan at a hospital. In Hardy's words:[76]

         > I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No", he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

         Immediately before this anecdote, Hardy quoted Littlewood as saying, "Every positive integer was one of [Ramanujan's] personal friends."[77]

         The two different ways are:

         $$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$
   h. Formula for pi