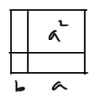
MA484 - Wk 8 - W

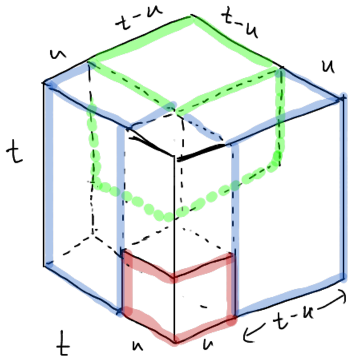
Cardano's Sol'n of the depressed cubic

$$\boxed{x^3 + mx = n}$$

Similar to geometric algebra



$= (a+b)^2$



block in back under green

$$t^3 = u^3 + (t-u)^3 + 2tu(t-u) + u^2(t-u) + u(t-u)^2$$

$$t^3 - u^3 = (t-u)^3 + (t-u)\underbrace{[2tu + u^2 + u(t-u)]}$$

$$\boxed{t^3 - u^3 = (t-u)^3 + (t-u)(3tu)}$$ universal truth

$s = x^3 - \frac{1}{x^3}$

$sx^3 = x^6 - 1$

$0 = x^6 - sx^3 - 1$

$w = x^3$

$0 = w^2 - sw - 1$

q.f.

$x^3 = w = \_\_$

write: $x = t - u$

$3tu = m$ (1)

$n = t^3 - u^3$ (2) $\Rightarrow t^3 - n = u^3$

(1) $u = \frac{m}{3t} \overset{\text{sub}}{\underset{(2)}{\rightsquigarrow}} n = t^3 - \left(\frac{m}{3t}\right)^3 = t^3 - \frac{m^3}{27t^3}$

mult. by t^3

$$nt^3 = t^6 - \frac{m^3}{27} \rightsquigarrow 0 = t^6 - nt^3 - \frac{m^3}{27}$$

$w = t^3$

$$\boxed{0 = w^2 - nw - \frac{m^3}{27}}$$

$$t^3 = w = \frac{n \pm \sqrt{n^2 + \frac{4m^3}{27}}}{2} = \frac{n \pm 2\sqrt{\left(\frac{n^2}{4} + \frac{m^3}{27}\right)}}{2} = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}$$

$$t = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}, \quad u^3 = n - t^3 = \frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3} - n = -\frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}$$

$$u = \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

$$\boxed{x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}}$$

Ex Solve $2x^3 - 7x - 2 = 0$ via Cardano

look like: $x^3 + mx = n$

$$x^3 - \frac{7}{2}x = 1 \qquad m = -\frac{7}{2} \qquad n = 1$$

$$x = \sqrt[3]{\frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{7}{6}\right)^3}} - \sqrt[3]{-\frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{7}{6}\right)^3}}$$

$$= \sqrt[3]{\frac{1}{2} + \sqrt{\frac{6^3 - 4 \cdot 7^3}{4 \cdot 6^3}}} - \sqrt[3]{-\frac{1}{2} + \sqrt{\frac{6^3 - 4 \cdot 7^3}{4 \cdot 6^3}}}$$

$$\sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-}$$

$$\left(\frac{-7}{2} \cdot \frac{1}{3}\right) = \left(\frac{-7}{6}\right)^3$$

$$\frac{1}{4} - \frac{7^3}{6^3} = \frac{6^3 - 4 \cdot 7^3}{4 \cdot 6^3}$$

Ex $9x^3 - 9x = 4 \longrightarrow x^3 - x = \frac{4}{9}$ $\quad m = -1 \; , \; n = \frac{4}{9}$

$$x = \sqrt[3]{\frac{2}{9} + \sqrt{\frac{4}{81} - \underbrace{\frac{1}{27}\cdot\frac{3}{3}}_{\frac{-3}{81}}}}$$

$$x = \sqrt[3]{\frac{2}{9} + \sqrt{\frac{1}{81}}} - \sqrt[3]{\frac{-2}{9} + \sqrt{\frac{1}{81}}}$$

$$x = \sqrt[3]{\frac{2}{9} + \frac{1}{9}} - \sqrt[3]{\frac{-2}{9} + \frac{1}{9}}$$

$$x = \sqrt[3]{\frac{3}{9}} - \sqrt[3]{\frac{-1}{9}}$$

$$x = \frac{3^{1/3}}{3^{2/3}} + \frac{1}{3^{2/3}} = \frac{3^{1/3}+1}{3^{2/3}} \cdot \frac{3^{1/3}}{3^{1/3}} = \frac{3^{2/3} + 3^{1/3}}{3}$$

$$= 3^{-1/3} - 3^{-2/3}$$