#3

(a) If $m \in \mathbb{Z}^+$, every prime factor of m occurs an even # of times in m^2.

let $m = p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}$

Then $m^2 = \left(p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}\right)^2 = p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m} \cdot p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m} = p_1^{2k_1} p_2^{2k_2} \cdots p_m^{2 \cdot k_m}$

Note p_i occurs $2k_i$ times & $2k_i$ is even.

(b) Suppose $\sqrt{2} \in \mathbb{Q}$. Then $\sqrt{2} = \frac{a}{b}$, w/ $a, b \in \mathbb{Z}^+$.

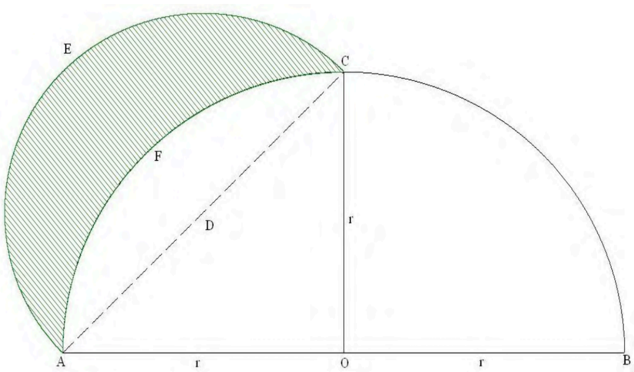
Squaring $\Rightarrow 2 = \frac{a^2}{b^2}$ or $a^2 = 2b^2$.

Since $a \in \mathbb{Z}$, a^2 has an even # of primes, in particular, an even # of 2's.

______ Also, $b \in \mathbb{Z}^+$, the same applies to b^2, so b^2 contains an even # of copies of 2. But then $2b^2$ must have an odd # of 2's. So a^2 has a factorization into an odd # of primes.

This contradicts the unique factorization theorem.

4.



(a) Area ΔAOC : $\frac{1}{2} r \cdot r = \frac{r^2}{2}$

(b) Area of F C D A : $\frac{1}{4}(\pi r^2) - \frac{r^2}{2}$

$= \frac{\pi r^2 - 2r^2}{4} = \frac{r^2(\pi - 2)}{4}$

(c) Area of E A D C : $\frac{1}{2}\pi\left(\frac{D}{2}\right)^2 = \frac{1}{2}\pi\left(\frac{D}{2}\right)^2 = \frac{\pi \cdot D^2}{8}$

$D^2 = r^2 + r^2$

$D^2 = 2r^2$

$D = \sqrt{2} \cdot r$

$= \frac{\pi \cdot 2r^2}{8}$

$= \frac{\pi r^2}{4}$

(d) Area of lune E A C :

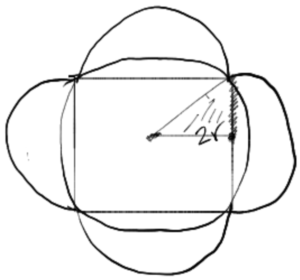
$=$ semi-circle (c) $-$ (b) $= \frac{\pi r^2}{4} - \frac{r^2(\pi - 2)}{4}$

$= \frac{\pi r^2 - \pi r^2 + 2r^2}{4}$

$= \frac{r^2}{2}$

(e) Triangle ΔAOC has the same area as the lune. Since this triangle is quadrable so is the lune.

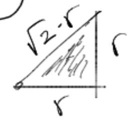
5.



(a) Total Semi-Circle Area

$$4 \cdot \frac{\pi r^2}{2} = 2\pi r^2$$

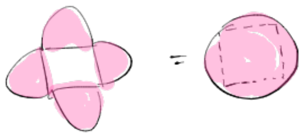
(b)



$\Longrightarrow$ Area of Circumscribed Circle

$$\pi(\sqrt{2}r)^2 = 2\pi r^2$$

(c)



so

Thus a single lune is quadrable b/c it has the same area as the orange square, which is constructable as an r by r square.