

Hint: usual congruence theorems apply proof by contradiction: Assume $D C \cong B A$

$\Longrightarrow$ SSS implies corr, angles $\cong$

$$
\begin{gathered}
\angle A B C \leftrightarrow<A D C \\
11 \\
90
\end{gathered}
$$

$\begin{aligned} \Rightarrow \beta & =90 \text { so } \triangle A B C D \text { does not } \\ & \text { satisfy HAA }\end{aligned}$

1. Form diagonal $D B$. Obviously $D B=B D$. If summits are equal, then SSS forces a summit angle to be 90, contradiction.
2. 

a) AAS
b) transitive property of equality on AH makes the heights equal.
C) the base has 4 sides but only two unique lengths, and DE contains one of each, so half
d) this is obvious, the two given angles overlap two angles of the triangle and the remaining part matches the third angle precisely

1. Since the triangle has the same angles as the two summit angles of a Saccheri, (who, by definition follows the HAA) are both acute) the triangle angles must be exactly acute + acute which is $<180$


If PT holds, then
... PT on biggest triangle gives ....
$B C^{\wedge} 2=(A E+C E)^{\wedge} 2+(A D+D B)^{\wedge} 2$
... algebra foil ....

$$
=\mathrm{AE}^{\wedge} 2+2 \mathrm{AECE}+C E^{\wedge} 2+A D^{\wedge} 2+2 A D D B+\mathrm{DB}^{\wedge} 2
$$

.... PT on smaller triangles and bisector assumption ...

$$
=2\left(E H^{\wedge} 2+A H^{\wedge} 2\right)+2 A E^{\wedge} 2+2\left(D H^{\wedge} 2+A H^{\wedge} 2\right)+2 A D^{\wedge} 2
$$

.... same idea

$$
=4\left(E H^{\wedge} 2+\mathrm{AH}^{\wedge} 2\right)+4\left(\mathrm{DH}^{\wedge} 2+\mathrm{AH}^{\wedge} 2\right)
$$

.... PT on triangle AED ....
$E D^{\wedge} 2=A E^{\wedge} 2+A^{\wedge} 2 \ldots$ then $P T$ again on smallest

$$
=\mathrm{EH}^{\wedge} 2+\mathrm{AH}^{\wedge} 2+\mathrm{DH}^{\wedge} 2+\mathrm{AH}^{\wedge} 2
$$

$$
\text { so } 4 E D^{\wedge} 2=4\left(E H^{\wedge} 2+\mathrm{AH}^{\wedge} 2\right)+4\left(\mathrm{DH}^{\wedge} 2+\mathrm{AH}^{\wedge} 2\right)
$$

thus $B C^{\wedge} 2=4\left(E D^{\wedge} 2\right)$ taking square roots gives
$B C=2(E D)$, but this implies the summits of a Saccheri are equal, contradiction"

