



Chapter 6

1. In what century did Cardano spend most of his life?
- ▼ 2. Give five reasons why Dunham writes that Cardano is the most bizarre mathematical character of all time.
 - ▼ a. Sadness
 - i. was illegitimate, almost aborted at birth, one son died, other jailed, wife died, impotent
 - ▼ b. Superstition
 - i. fear of heights, mad dogs,
 - ▼ c. Afflicted
 - i. massive urination, oozings, self-inflicted torture
 - ▼ d. Arrested
 - i. for heresy, casting horoscopes of Jesus
 - e. Pension from the Pope
 - ▼ f. Gambler
 - i. wrote a book on probability
 - ii. weaver of amulets
3. Briefly describe the Tartaglia / Cardano conflict.
- ▼ 4. How did complex numbers first find significance in mathematical thought?
 - a. As a tool for solving cubics.
- ▼ 5. What is Cardano's Great Theorem?
 - a. He produced a closed form solution for finding a real solution for many cubic equations.
6. Does there exist an algebraic formula for the quartic and quintic?

How to depress a poly. \longrightarrow remove $n-1$ term.

$$P(x) = x^5 + 5x^4 + 2x^3 + x + 1 = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

$n=5$

\downarrow (horizontal shift)

$$Q(x) = P\left(x - \frac{b}{n \cdot a}\right) = P\left(x - \frac{5}{5 \cdot 1}\right) = P(x-1)$$

$$= (x-1)^5 + 5(x-1)^4 + 2(x-1)^3 + (x-1) + 1$$

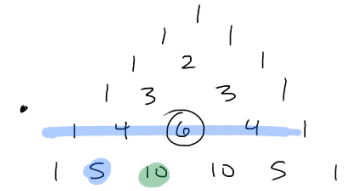
$$= x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

$$+ 5x^4 - 20x^3 + 30x^2 - 20x + 5$$

$$2x^3 - 6x^2 + 6x - 2 \Rightarrow Q(x) = x^5 - 8x^3 + 14x^2 - 8x + 2$$

$$x - 1$$

$$+ 1$$



- negative \Rightarrow signs alternate
- degree sum = constant (5)

Find a sol'n to _____

$$P(x) = x^3 - 3x^2 - 36x + 56 = 0$$

depress:

$$Q(x) = P\left(x - \frac{(-3)}{3 \cdot 1}\right) = P(x+1)$$

$$P(x+1) = (x+1)^3 - 3(x+1)^2 - 36(x+1) + 56 = 0$$

$$= x^3 + 3x^2 + 3x + 1 - 3x^2 - 6x - 3 - 36x - 36 + 56$$

$$Q(x) = x^3 - 39x = -18$$

ask Cardano _____.

$$x^3 + mx = n \implies \text{Cardano says}$$

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\left(\frac{n}{2}\right)^2 + \left(\frac{m}{3}\right)^3}}$$

$$\left(\frac{-39}{3}\right)^3 = -13^3$$

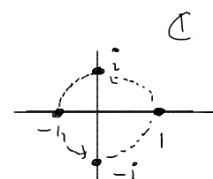
$$\sqrt{81 - 13^3} = 46i$$

$$= \sqrt[3]{-9 + \sqrt{81 - 13^3}} - \sqrt[3]{9 + \sqrt{81 - 13^3}}$$

$$i = \sqrt{-1}$$

$$i^2 = -1 \quad i^3 = -i$$

$$= \sqrt[3]{-9 + 46i} - \sqrt[3]{9 + 46i} \stackrel{\text{so}}{=} (3+2i) - (-3+2i) = 6$$



claim:

$$\sqrt[3]{-9+46i} = 3+2i \iff (3+2i)^3 = \boxed{-9+46i}$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

$$27 + 3(3^2)(2i) + 3(3)(2i)^2 + (2i)^3 = 27 + 54i - 36 - 8i = \boxed{-9+46i}$$

54i -4 -8i

$$Q'(6) = 0 \implies P(6+1) = P(7)$$

$$Q(x) = P(x+1)$$

Cardano

- autobiography - (superstitions, memories) (true?)
- Related to Pope Celestino II
- illegitimate, born after attempted abortion
- heart palpitations, fluids rozing, ruptures, hemorrhoids, peed 100 ounces/day
- fear of heights, impotence, insomnia
- Bite himself, twist fingers, self-inflicted torture (felt good when it stopped)
- studied @ U of Padua, couldn't get jobs in Milan (doctor)
- Practiced medicine in small towns
- Married woman of his dreams.
- Astrologer, weaver of amulets, had guardian angel, cast horoscopes
- Gambler: "Book of Chance", posthumously (1st book of probability)
- 1532 moved to Milan, lectures on popular science.
 - medicine, religion, math
- 1536: published an attack on corrupt practices of ^{Italian} doctors
- 1539: accepted in "College of Physicians"
- 1540s: most sought after doctor in Europe. (Pope)
- 1546: wife died, ^{leaving} son: Giambattista
- 1557: "wild woman", couldn't sleep, bed shook
 - bad omen, \Rightarrow Gi married a unworthy woman
 - Gi's wife had 3 children, none were Gi's, she flaunted infidelity, he killed her with cake, he's arrested, beheaded in 1560
- Cardano collapsed, lost everything (other son a criminal)
- 1562 \rightarrow left Milan for Univ. of Bologna, took Fabrizio, Gi's son.
- 1570 Cardano arrested for Heresy, casting horoscopes of deus wrote "In Praise of New", (anti-Christian Roman Ruler)

Somehow he got out (friends spoke), went to Rome
& got a pension from the Pope.

There as an old man, with "14 good teeth; one week" he
spent his last years until 1576.

- (7000 pages)

- modern, rational, yet superstitious (Middle Ages)

1st to systematically use negative numbers

1535 - Tartaglia bested Fior

Cardano asked T for details, refused, but eventually gave way.
(T sent Cardano the solution in cipher)

Cardano wrote to pledge faith that he'd keep secret.

Ludovico Ferrari (1522) asked for work, turned into
master/servant \rightarrow teacher/pupil \rightarrow colleague (Brilliant, young
profess^r)

Cardano shared secrets, the two made great progress
(general cubic: by reducing it to a depressed cubic.

Ferrari (did 4th degree) reduced 4th degree to related cubic.

- Two great discoveries, stymied.

1543: the two went to Bologna to study papers of Scipione del Ferro.
(30 yrs ago)

- they found solution to depressed cubic

- identical to Tartaglia's.

1545 - Cardano's Masterpiece: Ars Magna (Great Art)

- 40 chapters, XI: solution to Cubic
(preface: credited Tartaglia w/ the whole story)

Tartaglia: accused Cardano (remained silent)

but Ferrari (hot temper) lashed back w/ a nasty

1548 - Public Debate, Milan (Ferrari & Tartaglia)

\rightarrow lost, escaped alive

(nasty
pun
(roots
radishes))

complex, lusty, absurd