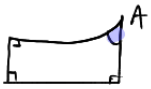
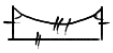
Week 10 - (Wed)
- assignment - proof presentation Pythag. Thm

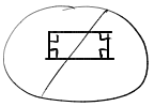
The Acute Angle Hypothesis: $\Longrightarrow$ implies



given a Lambert Quadrilateral, the remaing angle is acute

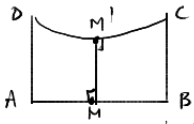
(1) Interior Angle α_i of Δ, $\sum_{\alpha_i \in \Delta} \alpha_i < 180°$ 

(2) Summit Angles of Saccheri Quad are $< 150°$ 

(3) Rectangles do not exist 

(4) Saccheri Summit $>$ Base (greater than)

(5) In a Saccheri Quad, segment joining midpoints (summit & base) is $\perp$ to both



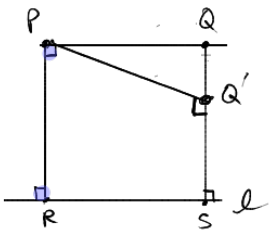
(6) In a Saccheri Quad, MM' is the shortest segment joining base & summit

(7) MM' is the only segment $\perp$ to both base & summit.

(8) Side adjacent the acute angle is greater than the opp. side.

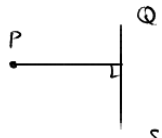
Thm: Universal Non-Euclidean Theorem:

Assume A.A.H. from above. $\forall$ lines ℓ, point $P \notin \ell$ $\exists$ at least two lines thru P, $\parallel$ to ℓ.

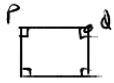


proof:

1. Drop $\perp$ from P to ℓ
2. Raise $\perp$ from P to ℓ (line $\perp$ PR), gives Q.
3. Corollary to AIA ($\Rightarrow$ $PQ \parallel \ell$)
4. Drop $\perp$ from Q to ℓ
5. Raise $\perp$ from QS to P



Is $Q' = Q$? If so then (No.)



$\Rightarrow$ rectangle (X) A.A.H.

6. $\overline{PQ'}$ & ℓ are cut by transversal $\overline{QS}$, & AIA's are $\cong$. AIA thm $\Rightarrow$ $\overline{PQ'} \parallel \ell$
7. $\overline{PQ'} \neq PQ$, so we have two lines thru P, $\parallel$ to ℓ.

Note: By varying point Q, we get multiple lines thru $P \parallel$ to ℓ.