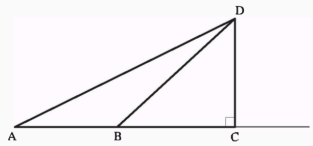
Happy - Week 11 [4 more lecture] + 1 week of finals

- Exercise p. 198, 22
- Exercises, p. 269, 1,3,4,8,9,10,11,12,15

p.198

22. Given $A * B * C$ and $\overleftrightarrow{DC} \perp \overleftrightarrow{AC}$. Prove that $AD > BD > CD$ (Figure 4.32; use Proposition 4.5).

(Katie)



Week 11 Assignment:

I assign you 1 problem this week to present.
the other problems are suggested (only).
Friday: you present sol'n to your exercise.
- You can trade w/ others if you like.

as last Friday
- prepare a way to write or slide or picture —

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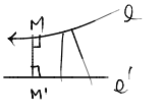
p.269 everyone try this!

#1 Statements equivalent to Euclid V
(Alan)

#3 (Boris)

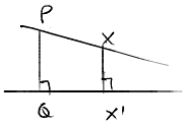


MM' is shortest segment b/w

#4 (Chloe)

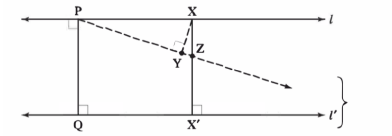
$AA' < BB'$

#8 (Tucker)



$PQ > XX'$

#9 (Davin)

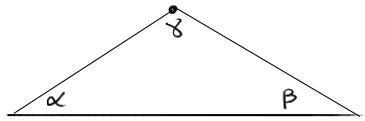


#10 (Euclidean) Proof of circumscribed Triangle
(Megan) - Use computer model to show it fails in $\mathbb{H}^2$.

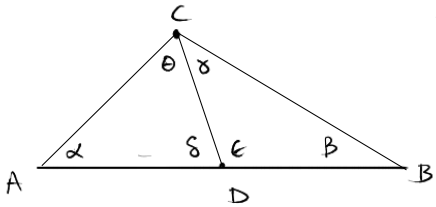
#11 Property of intersecting ⊥'s of △
(Hunter)

#12 Hyperbolic AIA
(Chris)

Defect of a triangle $\qquad D(\Delta) = 180^\circ - (\alpha + \beta + \gamma)$



Defect is additive: $\quad D(\Delta ADC) + D(\Delta DBC) = D(\Delta ABC)$



$$D(\Delta ABC) = 180 - \alpha - \beta - (\theta + \gamma)$$

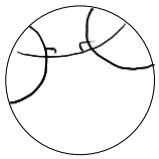
$$\left.\begin{aligned} D(\Delta ADC) &= 180 - \alpha - \delta - \theta \\ D(\Delta BDC) &= 180 - \beta - \epsilon - \gamma \end{aligned}\right\} \text{ add } \quad 360 - \alpha - \beta - (\theta + \gamma) - \delta - \epsilon$$

$$= -\overbrace{(\gamma + \epsilon)}^{180^\circ}$$

$$= 180 - \alpha - \beta - (\theta + \gamma) = D(\Delta ABC)$$

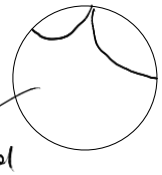
Defect of a Euclidean Δ is 0.

<u>Next</u>: Some properties of parallels that admit a common $\perp$.



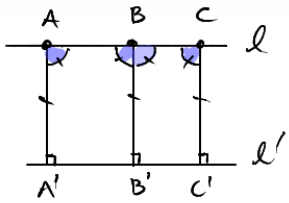
this!

not still parallel



$\mathbb{H}^2$

PROPOSITION 6.3. In a plane in which rectangles do not exist, if $l \parallel l'$, then any set of points on l equidistant from l' has at most two points in it, i.e., $\exists$ exactly two points on l that are equidistant from l'.



If there were 3 points of l equidistant from l', A, B, C, then if A', B', C' are feet of $\perp$ lines to A, B, C on l' then

$$AA' \cong BB' \cong CC'$$

$\square$ $A'B'BA$ is Saccheri, so summit angles are $\cong$. $\angle A'AB \cong \angle B'BA$

similarly

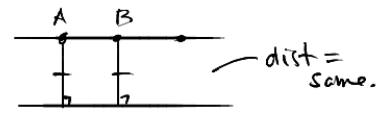
$\square$ $A'C'CA$ is Saccheri, so $\angle A'AB \cong \angle C'CB$

likewise

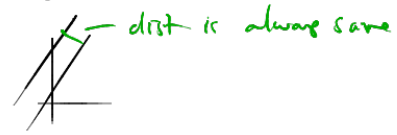
$\square$ $B'C'CB$ is Saccheri $\Rightarrow$ $\angle B'BC \cong \angle C'CB$

B/C $\angle B'BA$ & $\angle B'BC$ are supplementary & $\cong$ $\Rightarrow$ all angles are right $\otimes$.

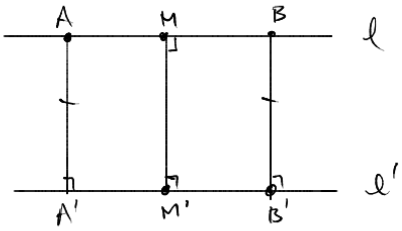
Compare to Euclidean Geom!

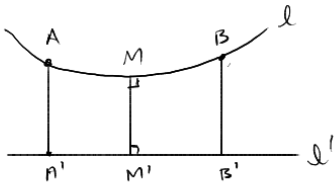


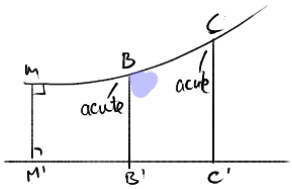
$y = x+1$ & $y = x+2$



PROPOSITION 6.4. In a Hilbert plane satisfying the acute angle hypothesis, if $l \parallel l'$ and if there exists a pair of points A and B on l equidistant from l', then l and l' have a unique common perpendicular segment MM′ dropped from the midpoint M of AB. MM′ is the shortest segment joining a point of l to a point of l', and the segments AA′ and BB′ increase as A, B recede from M.

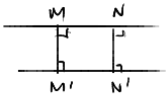






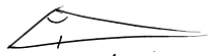
Previously, we've seen segment joining midpoints of base & summit are ⊥ to each:

Uniqueness comes from NO rectangles

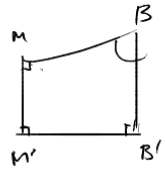


MM′ is shortest b/c:

we've proven



largest edge is opp largest angle ... also true for Lambert Quads (gen'l)



$\angle B'BM$ is <u>acute</u> then $\angle M'MB$ is the larger angle, thus $BB' > MM'$.

As B recedes away from M, it becomes say C

show $CC' > BB'$

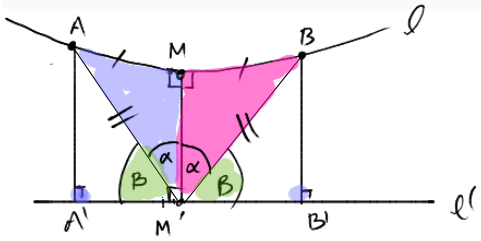
$\square M'B'BM$ & $\square M'C'CM$ are Lambert Quads in A.A.H.

supplementary: $\angle B'BM + \angle B'BC = 180$

$\Rightarrow \angle B'BC$ is <u>obtuse</u>

larger side is opp large angle

PROPOSITION 6.5. In a Hilbert plane in which rectangles do not exist, if lines l and l' have a common perpendicular segment MM′, then they are parallel and that common perpendicular segment is unique. Moreover, if A and B are any points on l such that M is the midpoint of AB, then A and B are equidistant from l'.



- By A1A, AA' is transversal to l, l', $l \parallel l'$.
- uniqueness by no rectangles

$AM \cong BM$ by hypothesis
$\angle AMM'$ & $\angle BMM'$ are right by hypothesis
$MM' = MM'$ (*)

$\Rightarrow \triangle AMM' \cong \triangle BMM'$

AAS-thm:

In $\triangle M'A'A$, $\triangle M'B'B$ we have AAS congruence ($AM' \cong BM'$ by (*))

$\Rightarrow AA' = BB'$