

In Hyperbolic Geometry: (A,A,H) : we understand a lot about \parallel lines that share a common \perp

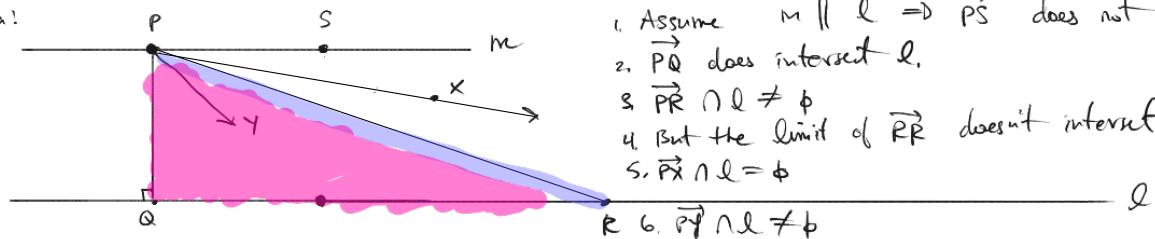
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- (#1) They exist (std. AIA construction)
 - (#2) MM' is unique (no rectangles)
 - (#3) \exists pts A,B equidist fr l'

Today: we talk about the other way lines can be \parallel .

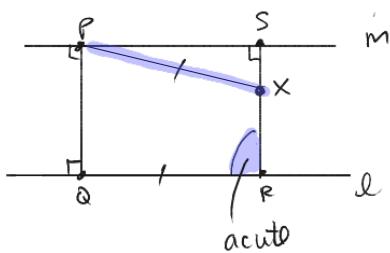
- ① No pair of points on l is equidistant to l' .
- ② No such \perp segment exists b/w parallel line $l \nparallel l'$.



Idea!



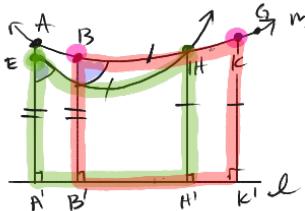
Saccheri



1. PQ is \perp to both $m \nparallel l$
2. $\square PQRS$ is Lambert Quad. in A,A,H
3. $\angle R$ acute $\Rightarrow PS < QR$
4. Find x s.t. $PX \cong QR$.
5. " \vec{PX} is the first ray which fails to meet l "

Classification of Parallels

Theorem 6.3: In a HT-plane w/ lines $l \parallel m$ st. m does not contain a limiting parallel ray, \exists common \perp to $l \nparallel m$



Proof: Choose any two points A, B on m , drop \perp to $A'B'$.
Assume $AA' > BB'$ (at most two pts equidistant)

Choose E on AA' st. $EN' \cong BB'$

Take $G \in m$, \nparallel transpose $\angle B'BG$ to E . The ray EG w/ this angle
does not intersect l .

Choose pt K on \vec{BG} st. $BK \cong EH$

Drop \perp 's fr $H \nparallel K$

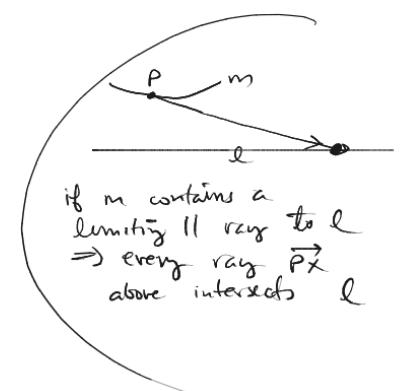
$$\square EA'H'H \cong \square BB'K'K \Rightarrow HH' \cong KK'$$

Saccheri's Dual.
Look at midpt. of Base/Summit
Get a common \perp to both

This says: If two lines are \parallel
they either share a common \perp or not

① If they do share a common $\perp \Rightarrow$ divergent \parallel .

② If they do not share a cm $\perp \Rightarrow$ asymptotic \parallel



Finding:
Heuristic argument showing "uniformity" - if \exists 1 acute angled quad \Rightarrow all quads are acute.

