

Henri Poincaré:

late 1800 -
early 1900's
1845-1912

Made profound discoveries in many branches of math / physics.

New branch of math: algebraic topology

Invented the fundamental group

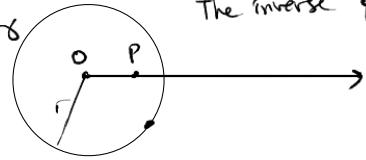
- Famous conjecture: 3-diml sphere was Millennium Problem

- Nearly, co-discovered theory of relativity

A study of Poincaré model requires "inversion circles"

Def'n: Inversion in circle γ .

The inverse of P is the point P' on \overrightarrow{OP} , s.t. $\overline{OP} \cdot \overline{OP'} = r^2$, r radius



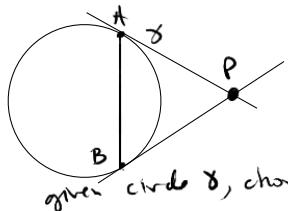
Prop 7.1: ① $P \in \gamma \Leftrightarrow \overline{OP} = r \Leftrightarrow \overline{OP} \cdot \overline{OP'} = r^2 \Leftrightarrow \overline{OP'} = r$ since $P' \in \overline{OP} \Leftrightarrow P = P'$

Inversion fixes the circle γ point-wise
in γ

② If P is inside γ then P' is outside γ . (& vice versa)
 $\overline{OP} < r$ then $\overline{OP'} > r$ in order for $\overline{OP} \cdot \overline{OP'} = r^2$
 P' is outside γ .

③ $(P')' = P$

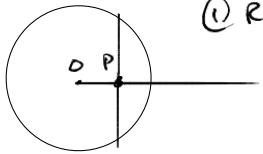
Def'n:



given circle γ , chord of γ , the pole of AB is P where $P = l \cap m$ w/ l tangent to γ @ A
 m tangent to γ @ B.

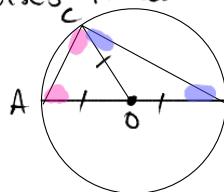
(Monday) Prop 7.2 the pole is used to compute inverse of P , (if P inside γ)

① Raise \perp to OP . P' is pole of this \perp segment.



Prop. 7.3 How to find interval of P , if P is outside δ .

Uses Thales' theorem:



Any Δ on diameter of a circle is right. (A Euclidean Result.)

① Total of isosceles Δ 's
Two

$$\angle A \cong \angle ACO, \angle B \cong \angle BCO$$

② Euclid \Rightarrow $\angle A + \angle ACO + \angle AOC = 180^\circ$
 $+ \angle B + \angle BCO + \angle BOC = 180^\circ$

③ $2\angle ACO + 2\angle BCO + 180^\circ = 360^\circ$

④ $\angle C = \angle ACO + \angle BCO = 90^\circ \quad \square$

Proof: ① Form OP , take midpt M ,

② circle $\odot M$, radius MP

③ Claim: $P = AB \cap OP$ is 'intervx.'

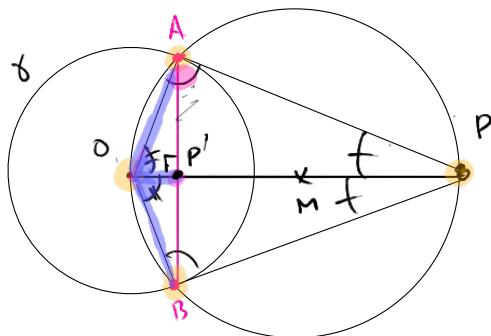
④ Thales $\Rightarrow \angle OAP \cong \angle OBP = 90^\circ$

⑤ $\begin{cases} \angle OAP = \angle OBP \\ \overline{OP} = \overline{OP} \\ OA = OB \end{cases}$ Hypo-Leg Crt $\Delta OAP \cong \Delta OBP$

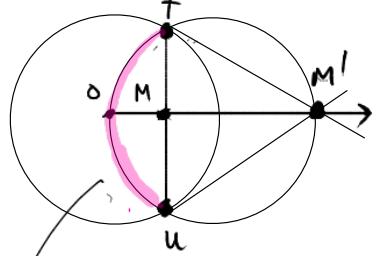
⑥ SAS $\Rightarrow \Delta OPA \cong \Delta OPB$
 $\Rightarrow \angle OPA' \cong \angle OPB = 90^\circ$

⑦ Now we apply Prop 7.2

{pole of AB is P' } P' is inverse of P'
 $\nparallel AB \perp OP'$ s. P' is inverse of P .



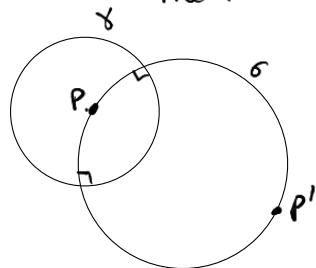
P-line = lines in Poincaré Model



Prop. 7.4

- ④ Given T, U (ideal points) find P-line b/w them.
- ① From TU , find midpt M , ray \overrightarrow{OM} . $\overrightarrow{OM} \perp TU$. M' = pole of TU is center of circle giving P-line
- ② this arc is the P-line

Prop. 7.5: Two circles, γ, σ meet orthogonally \iff the inverse of P in $\gamma \not\in P$ live in σ .



Assignment!

Use Prop 7.5 to : blow two given pts

- ① Construct P -line { blow two given pts in Poincaré Disk }
- ② Construct H -line { blow two given pts in Upper Half Plane } \Leftarrow



- ① Desmos.
- ② Talk through construction as you perform it
- ③ Either:
 - Record & share
 - < 5 min
 - Present live during office hours