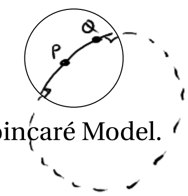
Study Guide: ______________________ **MA495 Exam 2** April 19, 2020

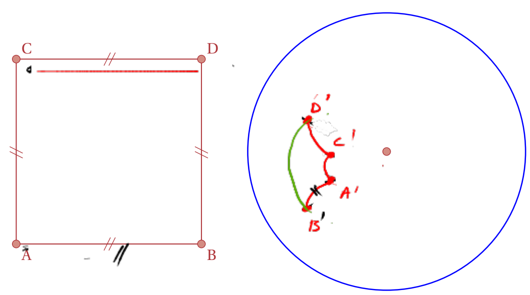
1. Note: P-model refers to the Poincaré model which is the usual (Euclidean) unit disk in $\mathbb{R}^2$.

# I Computations in Hyperbolic Geometry



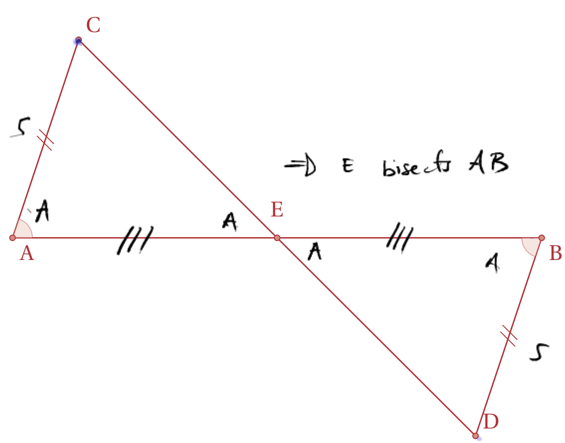

1. Briefly explain why the Incidence & Betweenness (& continuity) Axioms hold in the Poincaré Model.

2. Sketch the inversion of the square below about the circle.

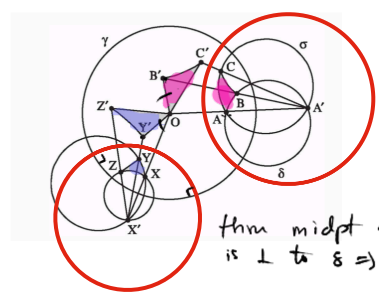



3. Breifly describe how inversions and rotations can be used to show that SAS congruence holds in hyperbolic geometry. (Work in the Poincaré model.) see p 327 (transpotion)

4. One way to construct the midpoint of a segment AB in Euclidean geometry is to choose a point C on one side of AB, mark of the angle ∡CAB and then form the congruent ∡ABD on the other side of the line AB as shown. The line CD then bisects AB. (Why?) Apply this same argument to hyperbolic geometry as follows:



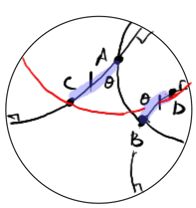

⇒ E bisects AB




then midpt of YY' is ⊥ to δ ⇒ Isometry

• we know inversions act transitively on Disk

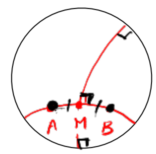
(a) Let AB be a P-segment in the P-model. Show how to construct its midpoint.



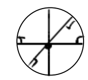

→


E is bisector

(b) Show how to find the perpendicular bisector of a segment in the P-model. By the preceding problem, all that remains is to describe how to find the perpendicular P-line to AB through a point M.

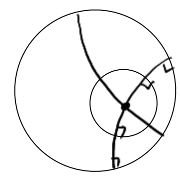



The p-line lies in the circle intersect boundary $\perp$, & inversion in this circle sends $A \longmapsto B$.



5. Show how to find the P-angle bisector of an angle in the P-model.

— Find segment bisector #4. Mimic constructions in Euclidean.



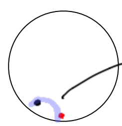
Intersection of lines $\perp$ to boundary gives center

$\mathbb{H}^2$

6. How does one construct the center of P-circle in the P-model?

7. Draw a picture in the P-model which describes Exercise 8, p. 272, and Exercise 9, p. 273.

Hint: Look up formula for a circle intersecting unit circle orthogonally.



$x^2+y^2+ax+by+1=0$ satisfies

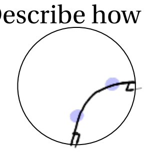
where this eqn intersects $x^2+y^2=1 \Rightarrow$ gives P, Q.

8. Let $(x_1, y_1) = (-0.85663050725, -0.515930396354)$ and $(x_2, y_2) = (-0.026468 - 0.999)$.

(a) Find the Euclidean distance between $(x_1, y_1)$ and $(x_2, y_2)$. $d(A,B) = \log|(AB, PQ)|$

(b) Find the Poincaré distance between $(x_1, y_1)$ and $(x_2, y_2)$. key: what is P, Q?

9. Describe how to find the P-line through points P and Q in the P-model.



10. What must be true if δ is a circle, R is an inverson about circle γ and R(δ) = δ? Intuitively, why must this be?

11. Is hyperbolic geometry consistent?

12. Does the existence of multiple lines through a single point which are all parallel to a given line suggest anything philosophically interesting to you?