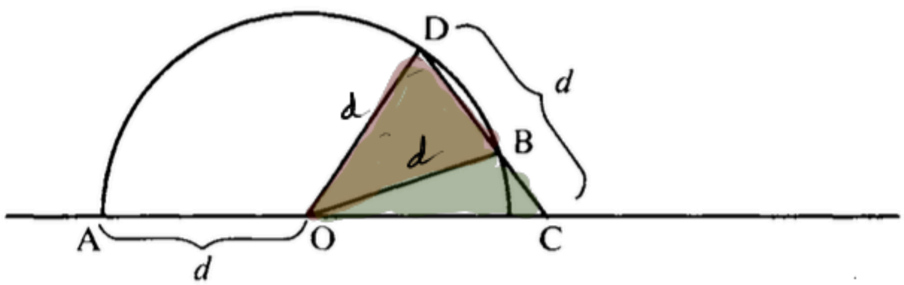
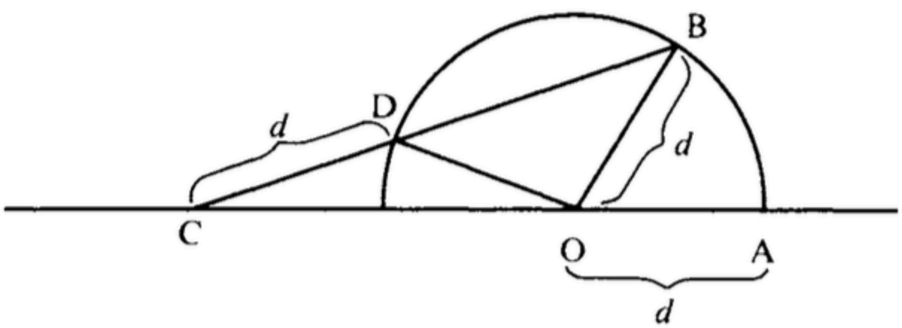
Archimedes Neusis Construction of trisecting angle.





$\angle AOB = \angle OCB + \angle OBC$ Ext. Angle

$= \angle DOC + \angle OBC$ isoceles

$\angle OBD + \angle OBC = 180$ supplementary angles

$\angle OBC = 180 - \angle OBD$ subtracting

$= 180 - \angle ODB$ isosceles

$= 180 - \angle ODC$ colinear

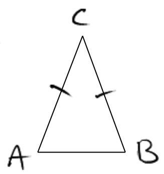
$= \angle DOC + \angle DCO = 2DOC$ Euclidean Δ's.

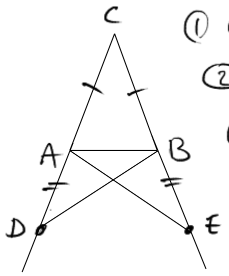
1. The Isosceles Triangle theorem

The angles oppossite equal sides of a triangle are congruent.

proof: (Look up "Bridge of Asses")

given





① extend CA & CB so that $AD \cong BE$

② Form segment DB & AE

③ Now $\triangle CDB \cong \triangle CEA$ by SAS ($\angle DCB = \angle CEA$)

④ So $\angle CDB \cong \angle CEA$ & $DB \cong AE$ by ③

⑤ Since $\angle ADE \cong \angle CDB$, $\angle BEA \cong \angle CEA$ & $AB \cong AB$ we conclude $\triangle ADB \cong \triangle BEA$ by SAS

⑥ Now $\angle DAB \cong \angle EBA$.

⑦ Since $\angle DAB + \angle BAC \cong \angle EBA + \angle ABC$

we have $\angle BAC \cong \angle ABC$

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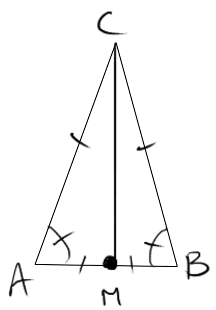
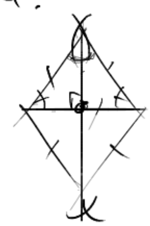
2. The $\perp$-bisector of the base in an isosceles $\triangle$ bisects the opposite angle.





① By Isosceles $\triangle$ thm $\angle CAB \cong \angle CBA$.

② Find the midpoint M of AB

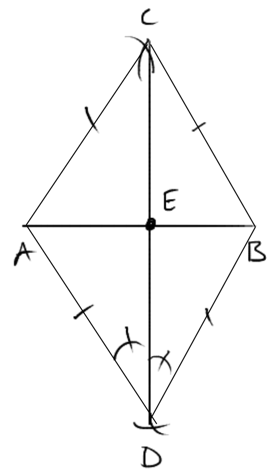
③ Form MC.

Now $\triangle AMC \cong \triangle BMC$ by SAS.

④ By ③ $\angle ACM \cong \angle BCM$ & $\angle AMC$ & $\angle BMC$ are congruent supplementary angles; thus are right.

⑤ CM is the $\perp$ bisector to AB & it bisects $\angle ACB$.

Construct the $\perp$-bisector of AB



① Intersect circles of radius AB centered @ A, B respectively to give points C, D

② Form CD

③ By construction $AC \cong BC \cong AD \cong BD$

④ By isosceles Δ thm $\angle CAB \cong \angle CBA$ so
$\angle DAB \cong \angle DBA$

$\angle CAD \cong \angle CBD$

⑤ By SAS $\Delta CAD \cong \Delta CBD$, thus $\angle ADC \cong \angle BDC$

⑥ By SAS $\Delta ADE \cong \Delta BDE$ since $DE \cong DE$

⑦ By ⑥ $\angle AED \cong \angle BED$ & are supplements so $\angle AED, \angle AEC, \angle BED, \angle BEC$ are all right angles
(supplement of right angle is right)

⑧ By ⑦ $AE \cong EB$ and CD is a $\perp$-bisector to AB