Incidence Geometry Axioms

(lies on)

(I.1) $\forall$ distinct points A, B, $\exists!$ line incident with them

(two pts give unique line)

(I.2) $\forall$ lines, $\exists$ at least two pts on it.

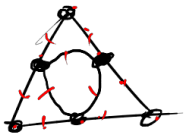
(I.3) $\exists$ 3 distinct non-collinear points

<u>Def.</u> 3 or more points are collinear if $\exists$ line incident w/ them all

<u>Def</u> 3 or more ~~points~~ lines are ~~collinear~~ concurrent if $\exists$ ~~line~~ point incident w/ them all

$\Rightarrow$ <u>duality</u> (in projective geometry — lines are <u>dual to</u> points.

See: major exercise #5



<u>Fano's Plane</u>. Finite Geom

Incidence Geometry - Geometry of (just) points & lines

Proposition 2.1. If l and m are distinct lines that are not parallel, then l and m have a unique point in common.

Proposition 2.2. There exist three distinct lines that are not concurrent.

Proposition 2.3. For every line, there is at least one point not lying on it.

Proposition 2.4. For every point, there is at least one line not passing through it.

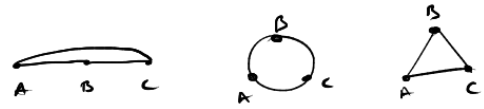
Proposition 2.5. For every point P, there exist at least two distinct lines through P.

— Model 1 —

Points = {A, B, C}

Lines = pairs

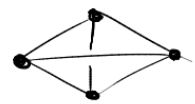
Elliptic //



— Model 2 —

Points = {A, B, C, D}

Hyperbolic //



— Model 3 —

Points = {A, B, C, D, E}

Hyperbolic // Property

Point:

Euclidean

the parallel property can not be proved from incidence axioms alone.