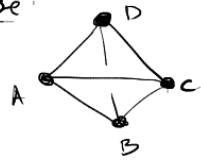
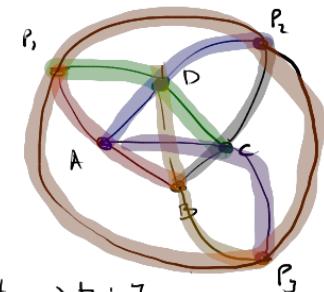


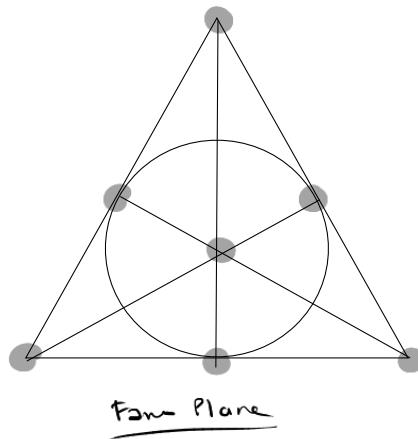
Exercise:



Projectively complete this affine plane.



points: 7
lines: 7



Algebraic Models of Affine Planes

$F = \text{field} : \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}_2^{\infty}$ $\{0, 1\} \cup \frac{1}{m} \mod 2$

$F^2 = \text{ordered pairs of elements of } F$. (algebra)

\mathbb{Z}_2 can be viewed as affine plane by:

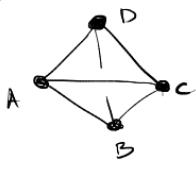
points = pairs

lines = solutions to $ax + by + c = 0$ (a, b, c not all zero)

Incidence: point lies on line if it solves the eqn.

(Affine Plane over the field F)

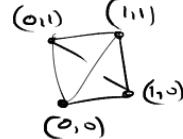
Ex. HW:



Field = \mathbb{Z}_2

$F^2 = \{(0,0), (0,1), (1,0), (1,1)\}$

$$ax + by + c = 0$$



$$\begin{array}{l} (0,1) \\ x=0 \\ y=1 \end{array}$$

$$a \cdot 0 + b \cdot 1 + c = 0$$

$$b + c = 0 \quad \text{work in } \mathbb{Z}_2.$$

$b = -c$, additive inverse of c . ($1+1=2 \mod 2=0$)

$$\begin{cases} a=0 \\ b=1 \\ c=1 \end{cases} \quad y+1=0 \quad (y=-1 \text{ but } 1=-1 \text{ here!})$$

$\Rightarrow (0,1)$ lies on $\begin{bmatrix} a, b, c \end{bmatrix}$

$$\begin{array}{l} (1,1) \\ x=1 \\ y=1 \end{array}$$

$$0 \cdot 1 + 1 \cdot 1 + 1 = 2 \mod 2 = 0 \Rightarrow (1,1) \text{ lies on } [0,1,1] \text{ too}$$

so $(0,1) \notin (1,1)$ determine $[0,1,1]$

Octonionization:

Field of numbers: Set w/ mult./add/ \nsubseteq inverses. ($\mathbb{R}, \mathbb{C}, \mathbb{Q}, \mathbb{Z}_p$)
Represent any of these fields by F . (algebra)

F^2 given structure of affine plane b/w:

(1) points:
lines: $ax+by+c=0$ (sols to this) = list, not
a continuing

I. (1) satisfies incidence axioms.

1. \exists 3 non-collinear points. $\{(0,0), (1,0), (0,1)\}$
 $a=0 \Rightarrow c=0$

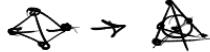
2. two points on every line: If $a \neq 0$, $(-\frac{c}{a}, 0)$ is on $x=0$ $\left(0, -\frac{c}{a}\right)$
 $b=0 \Rightarrow (-c/a, 1)$ is, $b \neq 0$ $y = -\frac{c}{b}$

$(u,v), (u_1, v_1)$ det. line

3. solve $\begin{cases} au+bu+c=0 \\ av+bv+c=0 \end{cases}$ } solve unique!

4. Eucl // Prop: Parallel \equiv disjoint, (\Leftrightarrow same slope). Given $ax+by+c=0$
 $\nparallel (u,v)$ not on it
Find line $\ell \ni (u,v) \perp l$

Last Time!



affine \Rightarrow projective
Plane - duality

Now, $F^2 \rightarrow P^2(F)$ = projective plane over F .

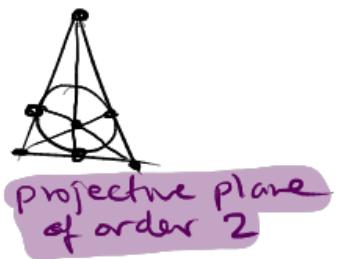
$(\mathbb{R}^2, \mathbb{Q}^2, \mathbb{C}^2, \mathbb{Z}_2^2)$

Week 4 - Wednesday

Last Time:

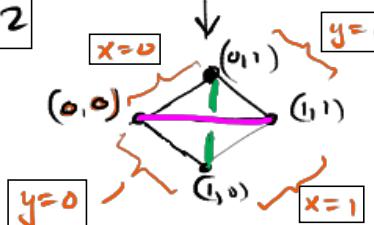


$\xrightarrow{\text{projective completion}}$



work mod 2

coordinatized
by $F = \mathbb{Z}_2$



pink line: $x+y=0$
green line: $x+y+1=0$

affine field over the field \mathbb{Z}_2