Prop. 3.3: $A*B*C$ & $A*C*D \Rightarrow B*C*D$ & $A*B*D$

Prop. 3.4: If $A*B*C$ w/ ℓ the line thru these pts, then $P \in \ell \Rightarrow P \in \overrightarrow{BA}$ or $P \in \overrightarrow{BC}$

Proof: ① Either $P \in \overrightarrow{BA}$ or not. If so, we're done. So assume $P \notin \overrightarrow{BA}$.

② then $A*B*P$. $(P*B*A)$

③ Regarding how B,P,C relate there are 3 possibilities:

(i) $B*P*C \Rightarrow P \in \overrightarrow{BC}$

(ii) $P*C*B \Rightarrow P \in \overrightarrow{BC}$

(iii) $C*B*P \Rightarrow$? . Assume this (RAA) $(P*B*C)$

④ Regarding A, C, P there are 3 possibilities

{8} (i) $P*C*A$: Combining w/ hypothesis $A*B*C \Rightarrow B*C*P \Rightarrow$ ⊗ step ④ (RAA)
$(A*C*P)$

{6} (ii) $C*A*P$: Combining this w/ step ② $P*B*A \Rightarrow B*A*C \Rightarrow$ ⊗ hypothesis
$(P*A*C)$

{7} (iii) $A*P*C$: Combining this w/ RAA-step ④ $\Rightarrow B*P*A \Rightarrow$ ⊗ step ②
$(C*P*A)$

⑤ Each sub-case of ② (iii) leads to a contradiction, so ② (iii) cannot hold

⑥ thus $P \in \overrightarrow{BC}$

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(1) $P \in \overrightarrow{BA}$
done!

(2) $P \notin \overrightarrow{BA}$

BW3 for B,P,C
- ① $P \in \overrightarrow{BC}$
- ② $P \in \overrightarrow{BC}$
- ③ P somewhere else

BW3 for A,P,C
- ① this + hyp $\Rightarrow$ ⊗ of 2.3
- ② this + 2.0 $\Rightarrow$ ⊗ of hyp
- ③ this + 2.3 $\Rightarrow$ ⊗ of 2.0